Physica A 471 (2017) 154–161

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Localized motion in random matrix decomposition of complex financial systems

Xiong-Fei Jiang [a,b,*], Bo Zheng [b,c,**], Fei Ren [d], Tian Qiu [e]

[a] School of Information Engineering, Ningbo Dahongying University, Ningbo 315175, PR China
[b] Department of Physics, Zhejiang University, Hangzhou, 310027, PR China
[c] Collaborative Innovation Center of Advanced Microstructures, Nanjing 210093, PR China
[d] School of Business, East China University of Science and Technology, Shanghai 200237, PR China
[e] School of Information Engineering, Nanchang Hangkong University, Nanchang 330063, PR China

HIGHLIGHTS

- The impacts of business sectors are identified from the localized motion.
- Localized motion induces different characteristics of the market index and stocks.
- Return-volatility correlations of eigenmodes are reproduced with a two-factor model.

ARTICLE INFO

Article history:
Received 14 August 2016
Received in revised form 7 November 2016
Available online 21 December 2016

Keywords:
Econophysics
Financial market
Business sector
Leverage effect
Two-factor model

ABSTRACT

With the random matrix theory, we decompose the multi-dimensional time series of complex financial systems into a set of orthogonal eigenmode functions, which are classified into the market mode, sector mode, and random mode. In particular, the localized motion generated by the business sectors, plays an important role in financial systems. Both the business sectors and their impact on the stock market are identified from the localized motion. We clarify that the localized motion induces different characteristics of the time correlations for the stock-market index and individual stocks. With a variation of a two-factor model, we reproduce the return-volatility correlations of the eigenmodes.

1. Introduction

Financial markets, as typical dynamic systems with highly complex interactions, share common features in various aspects with those in traditional physics. With large amounts of historical data, it becomes possible to analyze the complex financial systems based on concepts and methods in statistical physics [1–8]. Price movements of financial markets are characterized by the collective behaviors, such as the business-sector structure [9–11], leverage effect [2,12], extreme-volatility dynamics [3,13], and systemic risk [14–16].

The random matrix theory (RMT) is an important method to investigate the collective behaviors of complex systems, including financial markets. Based on the RMT theory, we develop the random matrix decomposition, with which the cross-correlation matrix can be decomposed into the mode cross-correlation matrices [17]. To further understand spatiotemporal

* Corresponding author at: School of Information Engineering, Ningbo Dahongying University, Ningbo 315175, PR China.
** Corresponding author at: Department of Physics, Zhejiang University, Hangzhou, 310027, PR China.
E-mail addresses: xfjiang@nbdhyu.edu.cn (X.-F. Jiang), zhengbo@zju.edu.cn (B. Zheng).

http://dx.doi.org/10.1016/j.physa.2016.12.021
0378-4371/

structures of complex financial systems, we decompose the multi-dimensional time series into eigenmode functions. In Particular, we investigate two typical spatiotemporal structures, i.e., the business-sector structure and return-volatility correlation.

The business-sector structure is not only crucial for theoretically understanding the complex structure of financial systems, but also important for optimal portfolio choice [18–20]. The RMT theory and network technique are two main approaches to identify the business sectors in the financial markets [17,21–28]. Besides, the emergence of collective behavior may also be described by the network synchronization [29,30]. However, both the RMT theory and network method cannot characterize the impact of business sector on the stock market, which is valuable for investors to hedge risk.

Time correlation functions describe how one variable is statistically influenced by another. For example, a negative return-volatility correlation, i.e. the leverage effect, is detected in almost all stock markets in the world [2,31]. Meanwhile, a positive return-volatility correlation, i.e. the anti-leverage effect, is observed in the Chinese stock market before the year 2000 [12,32,33]. However, this correlation is moderate for the individual stocks, while it is much stronger for the stock-market indices [2]. Therefore, the second motivation is to explore the complicated behavior of individual stocks in financial markets with the random matrix decomposition.

In this paper, with the RMT theory, we decompose the multi-dimensional time series of complex financial systems into a set of orthogonal eigenmode functions, which are classified into the market mode, sector mode and random mode. These three kinds of eigenmodes represent the global motion, localized motion, and quasi-random motion, respectively. The non-trivial interactions of financial systems are mainly dominated by the market mode and sector mode. Business sectors can be identified from the expansion coefficients of the individual stocks in the localized motion. More importantly, the positive or negative sign of the business sector describes the positive or negative impact of the business-sector motion on the stock market. In particular, the different magnitudes of the return-volatility correlations for the market index and individual stocks are rooted in the localized motion. We reveal that the leverage effect of the market index mainly originates from the global motion. Finally, a variation of a two-factor model is introduced to generate the return-volatility correlations of the three modes.

2. Method and basics

We define the logarithmic price return of the ith stock over an one-day time interval as

$$R_i(t) \equiv \ln P_i(t+1) - \ln P_i(t), \tag{1}$$

where $P_i(t)$ is the daily closed price of the ith stock at time t. To ensure different stocks with the same weight, the price return is normalized to

$$r_i(t) = \frac{R_i(t) - \langle R_i(t)\rangle}{\sigma_i}, \tag{2}$$

where $\langle \cdots \rangle$ is the average over time t, and $\sigma_i = \sqrt{\langle R_i^2(t)\rangle - \langle R_i(t)\rangle^2}$ denotes the standard deviation of $R_i(t)$. Then, elements of the cross-correlation matrix C is defined by

$$C_{ij} \equiv \langle r_i(t)\, r_j(t)\rangle. \tag{3}$$

Assuming N stocks with a total time length T, and in the large-N and large-T limit with $Q \equiv T/N \geq 1$, the probability distribution $P_{rm}(\lambda)$ of the eigenvalue λ for the Wishart matrix is given by

$$P_{rm}(\lambda) = \frac{Q}{2\pi}\frac{\sqrt{(\lambda_+ - \lambda)(\lambda - \lambda_-)}}{\lambda}, \tag{4}$$

where $\lambda_\pm$ are the upper and lower bounds given by

$$\lambda_\pm = \left[1 \pm \left(1/\sqrt{Q}\right)\right]^2. \tag{5}$$

We define the eigenmode function $k_\alpha(t)$ as

$$k_\alpha(t) = \sum_{i=1}^{N} u_i^\alpha r_i(t), \tag{6}$$

where u_i^α is the ith component in the αth eigenvector of the matrix C. The eigenmode functions form a set of orthogonal bases. For example, the market index may be expanded according to the orthogonal bases,

$$\phi(t) = \sum_\alpha B_\alpha k_\alpha(t). \tag{7}$$

The expansion coefficient B_α is given by

$$B_\alpha = \langle k_\alpha(t)\,\phi(t)\rangle, \tag{8}$$