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Stochastic effects in a discretized kinetic model of economic exchange[☆]

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HIGHLIGHTS

- We introduce random fluctuations in a discrete kinetic model for economic exchanges.
- We explore the effect of these stochastic uncertainties on wealth distribution.
- In the case of non-conserved total wealth we find positive correlation between Gini index and total wealth.
- In the case of conserved total wealth we find negative correlation between Gini index and economic mobility.
- The numerical results we obtain are in accordance with empirical evidence.

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ABSTRACT

Linear stochastic models and discretized kinetic theory are two complementary analytical techniques used for the investigation of complex systems of economic interactions. The former employ Langevin equations, with an emphasis on stock trade; the latter is based on systems of ordinary differential equations and is better suited for the description of binary interactions, taxation and welfare redistribution. We propose a new framework which establishes a connection between the two approaches by introducing random fluctuations into the kinetic model based on Langevin and Fokker–Planck formalisms. Numerical simulations of the resulting model indicate positive correlations between the Gini index and the total wealth, that suggest a growing inequality with increasing income. Further analysis shows, in the presence of a conserved total wealth, a simultaneous decrease in inequality as social mobility increases, in conformity with economic data.

1. Introduction

Microscopic models of economic interactions have been widely studied in the last years. The main goal has been an understanding of how a large number of monetary exchanges among individuals lead to certain income or wealth distributions and to specific values of global indicators like the Gini index or the economic mobility index, see e.g. [1] which contains a long list of references. In these models the economy of a country is seen as a complex system [2–4] and the Gini and mobility indices are emergent quantities; other macroscopic parameters summarizing the policies of governments, like for instance tax rates, welfare redistribution schemes etc., are introduced as input.

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Recent extensive work by Piketty and others [5–9] has highlighted the importance of economic inequality, raising questions about its natural evolution pattern in human societies, and consequently about when and how it is necessary for governments to intervene. Several empirical and theoretical studies have been devoted to the relation between growth and inequality, or to the question, as some put it, whether "a raising tide lifts all boats". The answer by Piketty, as is well known, contradicts the view of Kuznets and others [10,11].

Silva and Yakovenko [12] found in a case study that the fraction of the total population in the Pareto tail of the super-rich changes dramatically when the economy expands or contracts due to external interactions or variations in productivity. Some theoretical models allow to relate the total income of a society to the Gini index of its total income distribution. In [13], it was shown numerically that in a discretized kinetic model including taxation and redistribution, the Gini index of the equilibrium income distribution is an increasing function of the total income defined by the initial condition for a closed system. In a remarkable connection to statistical physics, it was also found [14] that the equilibrium income distribution can be well fitted by the κ-generalized distribution of Kaniadakis [15,16] which displays the same behavior as a function of the temperature and average energy (whereas the Gini index of the Boltzmann–Gibbs distribution is independent of them).

The discretized kinetic model also allows to determine a negative correlation between the Gini index G and economic mobility M which is confirmed by a large body of empirical evidence, nicknamed "the Great Gatsby law" [17,18]. The numerical solutions show [19] that when the model parameters (for instance, a parameter γ defining a means-tested welfare, and the tax rates τ_i) are varied at total fixed income μ, the corresponding variations of G and M are always of opposite sign, and such that it is possible to trace "level curves" for G and M in the $\gamma - \Delta\tau$ plane (with $\Delta\tau = \tau_n - \tau_1$). Like the relation between G and the total income μ, also this correlation has been established in the model as an equilibrium property.

Experience has also shown, however, that it is impossible to neglect stochastic factors in the evolution of economic systems, especially if strongly influenced by financial markets. As proven, among other instances, by the global crisis of 2008, random fluctuations can generate cascade failures and destabilize a system. The study of these phenomena was pioneered by W.B. Arthur in the 1980s and 1990s [3,4]. In conclusion, it is important to include stochastic aspects in the models. Life is full of randomness. While we head to our workplace in the morning, we can estimate the probabilities of many challenges or nuisances which are waiting for us; but we cannot predict those random disruptions that occasionally take us hostage for the whole day....

In this work we model ambient uncertainties by adding stochastic variations of the income distribution to the probability rates of change of the kinetic theory. In this way we are able to reproduce μ/G and M/G correlations which match those observed at equilibrium in the absence of noise. We regard this as a confirmation of general statistical properties which appear to apply to individuals in social science as well as to unanimated particles in the physical sciences.

The paper is organized as follows. In Section 2 we briefly recall first the evolution equations expressing the two complementary approaches of linear stochastic models and of discrete kinetic models; this then leads to a novel kinetic Langevin model. Numerical results for this model are discussed in Section 3, where also a Fokker–Planck equation for the case with additive noise is derived. The last section contains our conclusions and outlook.

2. Model formulation

2.1. Linear stochastic vs. kinetic model

In the formulation of mathematical models describing an ensemble of interacting agents who exchange money or assets one can choose, among others, between the two following schemes:

1. Consider N agents, with their individual wealth values w_i as fundamental variables; couple each agent to an external random source representing investment or stock trade and possibly also couple each agent to the others with a linear interaction term. The resulting stochastic equations can be solved numerically or transformed, in some cases, into a Fokker–Planck equation. This model, which we call "linear stochastic model", has been proposed by Bouchaud and Mezard [20]. Its mean-field approximation has also been independently obtained from the stochastic dynamics of a single agent in [21], and applied to an analysis of the poverty index in [22].
2. Re-group the N agents into n income classes, with $n \ll N$ and define a coupled dynamical system of the Boltzmann type which describes transitions between the classes. The fundamental variables are, in this case, the population fractions x_i ($i = 1, 2, \ldots, n$) of the classes. The interclass interactions are non-linear in these variables and the evolution equations fit into a discretized kinetic framework [13,14].

The two approaches are technically different and lend themselves to the analysis of different issues. Random fluctuations and trading are embedded from the start in the linear stochastic model. On the other hand, the discretized kinetic approach allows a more detailed description of the interactions and an analysis of effects like taxation, tax evasion [23] and welfare redistribution. The introduction of a network structure into kinetic theory has been discussed in [24] and compared with the analogous structure in the linear stochastic model [25]. Monte Carlo simulations [26,27] already constitute a bridge between the linear stochastic and kinetic approaches, because the variables of the simulations are the individual incomes of an ensemble of agents, but their binary interaction rules are of the kinetic type.

We recall next the equations of the linear stochastic model and of the discretized kinetic approach.