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Robustness of network controllability in cascading failure

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HIGHLIGHTS

- The measure of network joint cost and measure of controllable robustness are defined.
- The effect of control inputs and edge capacity on the controllability of Erdős–Rényi and Scale-free networks in cascading failure is studied.
- The change of controllability is apparently different in sparse and dense networks.
- Robustness of controllability will be stronger with less cost through increasing the number of input signals and edge capacity appropriately.

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ABSTRACT

It is demonstrated that controlling complex networks in practice needs more inputs than that predicted by the structural controllability framework. Besides, considering the networks usually faces to the external or internal failure, we define parameters to evaluate the control cost and the variation of controllability after cascades, exploring the effect of number of control inputs on the controllability for random networks and scale-free networks in the process of cascading failure. For different topological networks, the results show that the robustness of controllability will be stronger through allocating different control inputs and edge capacity.

1. Introduction

Complex network is a hot topic in recent decades, in which the nodes represent the individuals in real systems and the links capture the interactions among individuals [1–4]. Much efforts have been focused on the evolving of network structures and dynamics on complex networks, including traffic dynamics [5], synchronization [6], and evolutionary games on networks [7,8]. The very recent interesting has turned to explore the control of complex networks, which is also the ultimate goal of research about complex networks [9–12]. Then, an elementary step is understanding whether a system is controllable. A dynamical system is controllable if it can be driven from any initial state to any desired state with external inputs in finite time, according to the traditional control theory [13]. The nodes composed by external inputs are defined as driver nodes. Then, a critical issue is to find the minimal number of driver nodes to realize the full control of complex networks. A pioneer work of Liu et al. concludes that the minimal number of driver nodes to fully control the network is determined by the degree distribution of network [9]. Meanwhile, fixing the driver nodes of a directed network can also be transformed into the maximum matching. While the controllability of networks with arbitrary structures and link weights can be predicted by exact controllability theory [12]. In addition, numerical controllability [14], target control [15], control profiles [16] and other researches have also been explored [17–26].

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It is necessary for us to concern the robustness of complex networks, because the networks usually suffer from attacks, that may be random or intentional, can lead to the failure of nodes and links [27–31]. Then, the redistribution of load may spread the failure and causes cascading failures. A networked system, that needs less driver nodes to achieve full control at the very beginning should increase the number of driver nodes to full control, when the system suffers from cascading failure [32–35]. Here, we introduce the control cost, which is defined as the function of number of driver nodes before cascading and tolerant parameter of edge capacity, exploring the controllability of networks in the process of cascading failure. Simulation results show that, tolerant parameter and average degrees of networks can greatly impact the increment of number of driver nodes. Networks with sparse or dense links exhibit strong robustness of controllability, because the cascading failure is restrained. This paper is organized as follows: Section 2 presents the model, Section 3 demonstrates numerical results, and the discussion is presented in Section 4.

2. Models

Load model of cascading failure. Each link e_{ij} at step t in network is assigned load $B_{ij}(t)$, which is set as the total number of shortest paths in network passing through the link e_{ij}, and $B_{ij}(0)$ is initial load. The capacity C_{ij} that a link can handle the maximum load is defined as the following [27]:

$$C_{ij} = \alpha B_{ij}(0), \tag{1}$$

where α is the tolerant parameter. The link e_{ij} is failed as the load $B_{ij}(t)$ exceeding its capacity C_{ij}, and it will be removed from the networks. Then, the load of e_{ij} will redistribute.

Controllability of networks. The dynamical equation of a linear time-invariant system with N nodes can be described as the following:

$$\dot{x} = Ax + Bu, \tag{2}$$

where the vector $x(t) = [x_1(t), x_2(t), \ldots, x_N(t)]$ is the state of N nodes at time t. $A = (a_{ij})_{N\times N}$ is the adjacent matrix and input matrix $B = (b_{ij})_{N\times M}$ defines how the input signals are imposed to the nodes of networks. $u(t) = [u_1(t), u_2(t), \ldots, u_M(t)]^T$ is the input vector. The system (A, B) defined by Eq. (2) is controllable if the controllability matrix $C = (B, AB, \ldots, A^{N-1}B)$ has full rank [36]. Liu et al. found that the minimal number of inputs or driver nodes N_D for a directed network is determined by degree distribution, and determining the set of driver nodes can be transformed into maximum matching [9]. At the beginning, N_D nodes are selected by structural controllability and $N_I - N_D$ nodes are randomly chosen as the driver nodes. We define a parameter θ to capture the function of N_I and N_D:

$$n_I = \frac{N_I}{N} = \frac{\theta N_D}{N} = \theta n_D, \tag{3}$$

where n_D is the fraction of minimum driver nodes. The control cost is defined as: $T = \alpha e^{n_I}$. Once the link with largest load is removed, the minimum fraction of driver nodes to full control the network increases caused by cascading failure. We define Φ as the network robustness of controllability, to characterize the increment of control inputs before and after the cascading failure:

$$\Phi = \frac{N_D^f - N_I}{N}. \tag{4}$$

3. Results

We explore the network controllability in the process of cascading failure, as shown in Fig. 1. The random networks with lower (sparse) or large enough (dense) average degrees $\langle k\rangle$ show stronger robustness of controllability, where the increments of number of driver nodes are less. The removal of link with largest load in sparse network renders some nodes to be unreachable, then the loads of links are reduced, which sustains the spreading of failure. Hence, controllability of networks after the cascading failure is similar to that before failure. While for random networks with moderate average degrees, increments of number of driver nodes show a sharp rising at the critical time step $T = 4$, since the failure is widely spread and large amount of links are failed. Then, the number of driver node n_D should be increased to full control the networks. Fig. 1(b) shows controllability of scale-free networks with different average degrees in the process of cascading failure. The dense scale-free network, i.e. $\langle k\rangle = 20$ exhibits weak robustness of controllability, because of degree heterogeneity. Fig. 1 demonstrates that, increments of driver nodes of network controllability is closely related with the amount of failed links in cascading failure.

However, with the consideration of energy consumption and control trajectory, it usually needs more external inputs to full control the whole system, in which the number of driver nodes is larger than the minimal driver nodes N_D. Therefore, based on the control cost T before and after cascades, we explore the change of controllability in cascading failure. As shown in Fig. 2, we illustrate the change of controllability with different control cost in random networks with varied average degrees. In Fig. 2, with the lower $\langle k\rangle$, the change of controllability is almost the same with the increasing of edge capacity