Crowd of individuals walking in opposite directions. A toy model to study the segregation of the group into lanes of individuals moving in the same direction

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HIGHLIGHTS

- We analyze a mathematical model of the formation of lanes in crowds.
- We prove that the system self-organizes.
- We compute the time required for self-organization.

ARTICLE INFO

ABSTRACT

Consider a corridor, street or bridge crowded with pedestrians walking in both directions. The individuals do not walk in a completely straight line. They adjust their path to avoid colliding with incoming pedestrians. As a result of these adjustments, the whole group sometimes end up split into lanes of individuals moving in the same direction. While this formation of lanes facilitates the flow and benefits the whole group, it is believed that results from the actions of the individuals acting only on their behalf, without considering others. This phenomenon is an example of self-organization.

We analyze a simple model. We assume that individuals move around a two-lane circular track. All of them at the same speed. Half of them in one direction and the rest in the opposite direction. Each time two individuals collide, one of them moves to the other lane. The individual changing lanes is selected randomly. The system self-organizes. Eventually each lane is occupied with individuals moving in only one direction. We show that the time required for the system to self-organize is bounded by a linear function on the number of individuals. This toy model provides an example where global self-organization occurs even though each member of the group acts without considering the rest.

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1. Introduction

Self-organization refers to the spontaneous appearance of order in systems composed of several units. Frequently, self-organization is not planned. It is observed even when each unit acts on its behalf, without considering the whole group. A main interest of researchers is to understand how local interactions among units and with the environment lead to self-organization. Recent books and review articles on self-organization in biological systems include [1–4].

Self-organization is sometimes observed in crowds of walking humans. For example, assume two rooms are connected by a closed door and a large number of individuals is in each room. Suppose each individual wants to move to the room they

are not in. If the door is opened, the spontaneous formation of an alternating flow is sometimes observed. Individuals from only one of the rooms cross the door for a period of time and then there is a sudden switch, individuals from the other room are the only ones crossing the door after the switch. This switching or turn taking persists while the environment remains crowded [5].

The formation of trails is a second example of self-organization due to walking individuals. Assume individuals need to cross, possibly in different directions, the same mildly dense grass field on a regular basis. Individuals will try to walk along the areas where the grass is shorter and less dense, even if the path they take as a result is not the shortest. Stepping on the grass is how the individuals affect the environment. Grass has a harder time growing in the areas heavily transited. As a result, dirt path trails form [6].

A third example of self-organization is the different pattern formations observed when the crowd is a collection of several smaller groups. For example, these smaller groups may be groups of friends or families. We refer the reader to [7] for more details.

In the described examples, the coordinated behaviors observed are not planned by the crowd as a whole or by any member or group of members within the crowd. Instead, they emerge spontaneously, as a consequence of the individuals acting in response to local stimuli and motivated by their own goals.

The dynamics of crowds is a source of examples of self-organization and may lead to strategies to increase the safety in crowded areas such as bridges [8] or stadiums. It can provide guidelines in the design of movie theaters, shopping malls, or other similar type of heavily transited buildings, where optimal crowd flow is desired because of economical and safety reasons. Accordingly, the study of dynamics of crowds, both theoretically (early works include [9,10], see also [3,11] for reviews) and experimentally [12–14], is a very active area of research.

Microscopic mathematical models to study dynamics of crowds are those that keep track of the position and the velocity of each individual. Cellular automata models [15–17], lattice gas automata models [18,19], algorithms [20], and large systems of odes, are all examples of microscopic models. When large systems of odes are used, the mass times acceleration of each individual is set equal to the sum of *generalized or social forces* the individual feels [21–25]. These *social forces* are not real forces. They model the responses of the individuals to the environment and the presence and actions of the other individuals [26]. Some of these models are known as self-propelled particles models [27] and others as individual based models [26,28].

Mesoscopic, kinetic or Boltzmann type models to study the dynamics of crowds, are integro-partial differential equations that describe the evolution of probability densities of the position and velocities of the individuals [29–31]. Macroscopic or continuum models are partial differential equations (conservation equations), where the dependent variables are the density and *local average* velocity of individuals [32–36]. Some works connect microscopic to macroscopic models [37–39].

Assume a corridor, street or pedestrian bridge is crowded with persons walking. Some of the pedestrians walk in one direction while the others in the opposite direction. A self-organizing phenomenon frequently observed is that the individuals segregate into lanes of individuals moving only in one direction [40–42]. Needless to say, this formation of lanes benefits the whole group, as it results in an easier flow in both directions [43].

Motivated by the phenomenon described in the last paragraph, in [44] we introduced a new mathematical model. Briefly, individuals move around a two-lane circular track with the same angular speed. Half of them walk clockwise and the rest counterclockwise. Each individual remains in its lane unless it collides with an other individual walking in the same lane but in the opposite direction. When such a collision occurs, one of the colliding individuals moves to the other lane. The individual changing lanes is selected randomly. In [44] we showed that, with probability 1, the system *self-organizes*, i.e. eventually all the individuals moving counterclockwise end up in the same lane and all the individuals moving clockwise end up in the other lane. In this article, we prove that the expected time required for the system to self-organize is bounded by a linear function of the number of pedestrians. This explains numerical findings in [44].

This article is organized as follows. In Section 2 we describe the model and show the results of some numerical simulations. In Section 3 we show the results of numerical simulations that strongly suggest that the expected time to self-organization is asymptotically a linear function of the number of individuals. In Section 4 we obtain a recurrence relation that governs the dynamics of the system. Our analysis is done in Sections 5–8. We finish with a short discussion in Section 9.

2. The model

$2N$ individuals or pedestrians circle around a two-lane circular track. Half of the individuals move in the clockwise direction and the other half in the counterclockwise direction. Each time two individuals moving in opposite directions and in the same lane meet, we say they collide. When two individuals collide exactly one of them, randomly chosen, with each having the same probability of $1/2$ of being chosen, moves to the other lane.

Next, we enumerate a list of statements. The *i*th statement will be referred as Statement *i* from Section 2. These statements are either rules that help precisely define the dynamics of our system, or observations that are consequences of those rules and will be needed in the analysis in subsequent sections.

1. $2N$ individuals move around a circular track with 2 lanes. Each lane is labeled with a number. The inner lane is lane 1 and the outer lane is lane 2.
2. N individuals move in the counterclockwise direction and the other N in the clockwise direction.