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Drag force in wind tunnels: A new method

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HIGHLIGHTS

- We introduce a new numerical method to calculate the drag force over an object in a viscous media.
- We confirm the experimental observation that a falling ball can lose speed.
- We can describe the drag force during the transient regime.

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ABSTRACT

A rigid object of general shape is fixed inside a wind tunnel. The drag force exerted on it by the wind is determined by a new method based on simple basic Physics concepts, provided one has a solver, any solver, for the corresponding dynamic Navier–Stokes equation which determines the wind velocity field around the object. The method is completely general, but here we apply it to the traditional problem of a long cylinder perpendicular to the wind.

1. Introduction

Consider a rigid object facing the counter flux of a viscous fluid (air or water, for instance). The fluid velocity field around the object (which would be uniform in its absence) is distorted by its presence. This velocity field $\vec{v}(\vec{r})$ can be obtained as a function of time t by solving the so-called Navier–Stokes dynamic equations [1,2]. These equations can be written in different forms, here we adopt a simple one

$$\frac{\partial \vec{\Omega}}{\partial t} = \frac{1}{Re}\nabla^2\vec{\Omega} - \vec{\nabla} \times (\vec{\Omega} \times \vec{v}), \tag{1}$$

see for instance [3], where

$$\vec{\Omega} = \vec{\nabla} \times \vec{v} \tag{2}$$

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is the vorticity field. Also, provided all speeds are small compared with the sound propagation speed in the same fluid, it can be considered incompressible, *i.e.*,

$$\vec{\nabla} \bullet \vec{v} = 0. \tag{3}$$

Eq. (1) is already expressed in adimensional units, considering the wind speed V far from the object and some characteristic linear dimension of the rigid object such as the diameter D of a cylinder, for instance. Together the characteristic fluid properties ρ, its uniform density, and μ, its viscosity, these parameters can be condensed into a single adimensional Reynolds number [4]

$$Re = \frac{\rho V D}{\mu}. \tag{4}$$

This way, one can consider V in Eq. (1) as the speed unit, and D as the length unit. The corresponding time unit is D/V.

These 4 above equations determine completely the velocity or vorticity fields around the object, as functions of time, given the proper boundary and initial conditions. Normally, the boundary condition is $v = 1$ far from the object along some fixed direction, and $v = 0$ at the object surface. The initial condition may be (as here) the wind tunnel initially switched off and suddenly switched on with a fixed value V (or Re, impulsive switching). In this case, one has first to solve the time independent Stokes equation $\nabla^2\vec{\Omega} = 0$, obtained by taking $Re \to 0$ in Eq. (1), in order to obtain the initial fields $\vec{v}(\vec{r})$ and $\vec{\Omega}(\vec{r})$ at $t = 0$. By neglecting this care with the initial fields, one obtains spurious transient fields just after $t = 0$, while the steady-state final regime is not reached [5]. Nevertheless, it is important to emphasize that Eq. (1) is valid for any Re.

What is not explicit in the above equations is how to determine the drag force the wind exerts on the fixed object. In general, the drag force may be obtained by two different types of methods. There are extrinsic methods, where the force exerted by the fluid on the body may be evaluated directly from the external force needed to hold the body onto the given trajectory (see, for example, [6]), and intrinsic methods, where the force exerted by the fluid on the body may be derived from the equations of fluid mechanics and evaluated using measured flow-field quantities. An intrinsic method surpasses an extrinsic technique, for example, by its ability to measure sectional forces and small force levels [7].

There are several intrinsic approaches [7–13] but, the traditional intrinsic way to do this is by first determining the pressure distribution and the shear stresses and then integrating them on the object surface [14]. Alternatively, one can obtain the drag force just through the knowledge of the fluid velocity field around the object. In both cases, an integration on the surface of the object that involves the gradient of this field on the surface is necessary [7,15]. If the method adopted to solve the above equations is a numerical one on a discrete grid, the determination of the quoted gradient requires a very fine grid near the surface, which takes a large computational effort. That is why normally researchers adopt non-uniform grids, very fine only near the object surface, not only in order to enhance the precision in the boundary layer where the smaller speeds require finer grids, but also when the aim is to determine the drag force. This approach unnecessarily complicates the mathematical discretization procedures needed to translate the continuous derivatives into finite differences, besides the further computer effort fine grids require.[1]

The current text introduces an alternative method to determine the drag force replacing the surface integration by a volume integration, which solves both problems commented at the end of last paragraph. The method is completely general and based only on simple basic Physics concepts. It is described below, and its validity mathematically demonstrated at Appendix (although this mathematical demonstration is completely unnecessary, the conceptual arguments below are enough).

Consider one has a solver for the Navier Stokes equations above, providing the field $\vec{v}$ in all space at time $t + \delta t$ from the knowledge of the previous field at time t, where δt is some small time interval. Any solver can be used, the accuracy of the method now described depends only on the accuracy provided by this solver. Imagine one replaces the rigid object by an extra portion of fluid at time t. This extra portion of fluid is also at rest in the same way as the removed object. But it would move from time t on. A small fluid velocity would appear at each point inside the volume formerly occupied by the object, at $t + \delta t$. By integrating these velocities inside the quoted volume and multiplying the result by the density of the fluid, one obtains the total linear momentum which would be transferred from the fluid to the object, a vector. This would-be transferred momentum is compensated by the mechanical device keeping the object at rest inside the wind tunnel. Finally, dividing this total linear momentum by δt, one obtains the force exerted by the original fluid on the object.[2]

In short: (I) One has the velocity field at time t around the object, with all other velocities null inside the volume formerly occupied by the object, now occupied by the static fluid replacing it; (II) One uses the Navier Stokes equations solver in order to obtain the same field at $t + \delta t$; (III) Non-null, small velocities appear inside the quoted volume; (IV) One integrates these penetrating small velocities over the quoted volume, multiplies the result by the fluid density and divides it by δt.

[1] Indeed, fine grids are required not only to determine the gradient but to properly solve the boundary layer (a layer, usually very thin, near any fixed surface in a moving stream in which shearing stresses are not negligible) and describe the general pattern of the flow. This is true even for the proposed method in this paper. However, using our method, no change in density of the grid was performed to properly solve the boundary layer and even so, we obtained forecasts according to experimental data.

[2] The time δt in question may be chosen arbitrarily. The choice is acceptable only if the following condition is satisfied: when the time δt is multiplied by two, the transferred momentum becomes twice as large. This condition may be attended independent of the time step used in the method to solve the Navier–Stokes equation. In all of our tests, the δt adopted is the same as the discrete time interval used to solve the Navier–Stokes equation, i.e. the time the wind takes to transverse 0.1 grid pixel.