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# The application of the multifractal cross-correlation analysis methods in radar target detection within sea clutter

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## HIGHLIGHTS

- We study the parameter settings and the applicabilities of MFXPF, MFXDFA and MFXDMA.
- Do comparative analysis of the artificial time series with different probabilities by these methods.
- Provide a guideline on the choice and parameter settings of the three methods in practice.
- Feasibility test of the target detection method based on the multifractal cross-correlation analysis.
- Summarize the flow chart of radar detecting low-observable target within sea clutters.

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## ABSTRACT

Many complex systems generate multifractal time series which are long-range cross-correlated. This paper introduces three multifractal cross-correlation analysis methods, such as multifractal cross-correlation analysis based on the partition function approach (MFXPF), multifractal detrended cross-correlation analysis (MFDCCA) methods based on detrended fluctuation analysis (MFXDFA) and detrended moving average analysis (MFXDMA), which only consider one moment order. We do comparative analysis of the artificial time series (binomial multiplicative cascades and Cantor sets with different probabilities) by these methods. Then we do a feasibility test of the fixed threshold target detection within sea clutter by applying the multifractal cross-correlation analysis methods to the IPIX radar sea clutter data. The results show that it is feasible to use the method of the fixed threshold based on the multifractal feature parameter $\Delta f(\alpha)$ by the MFXPF and MFXDFA-1 methods. At last, we give the main conclusions and provide a valuable reference on how to choose the multifractal algorithms, the detection parameters and the target detection methods within sea clutter in practice.



## 1. Introduction

There are a number of situations in which several signals are simultaneously recorded in complex systems, which exhibit long-term power-law cross-correlations [1]. A variety of methods have been used to investigate the long-range power-law cross-correlations between two nonstationary time series [2]. The earliest is joint multifractal analysis based on

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the partition function approach with two moment orders to study the cross-multifractal nature of two joint multifractal measures through the scaling behaviors of the joint moments [2,3], which is a multifractal cross-correlation analysis based on the partition function approach (MFXPF $(p, q)$) [2]. Then MFXPF $(q)$ method [4,5] which only considers one moment order has been proposed independently and developed into weighted MFXPF (WMFXPF) [6]. The MFXPF $(q)$ method reduces to the Qth order moment structure partition function (QMSPF) method [7] when $\{X(i)\} = \{Y(i)\}$. The Qth order mixed moment structure partition function (QMMSPF) method [5], which is a method based on multifractal cross-correlation analysis, has been developed from the QMSPF method. But the $q$-order moment structure function $Z_{xy}(q, s) = \sum_{v=1}^{N_s} \left|p_{xv}(s)p_{yv}(s)\right|^q, q \in \mathbb{R}$ is different from the degeneration of the statistic moment function $Z_q(s) = \sum_{v=1}^{N_s} \left|p_{xv}(s)p_{yv}(s)\right|^{q/2}, q \in \mathbb{R}$ in the MFXPF $(p, q)$ method with the joint moments $p = q$ [6]. Over the past decade, detrended cross-correlation analysis (DCCA) has become the most popular method of investigating the long-range power-law cross correlations between two nonstationary time series [8], and this method has numerous variants [9–16]. There is also a group of multifractal cross-correlation analysis methods of analyzing multifractal time series [1,16–22], such as multifractal detrended partial cross-correlation analysis (MFDPXA) [16], multifractal Fourier detrended cross-correlation analysis [17] and multiscale multifractal detrended cross-correlation analysis [18–20], MFXDFA [1], MFXDMA [1] and multifractal height cross-correlation analysis (MFHXA) [22]. Note that it makes sense to analyze the scaling only for the two detrended series and only for $q > 0$ by MFHXA [22]. In this paper, we compare the algorithm models and their requirements in practical applications.

The expected values of the bivariate Hurst exponents have been partly discussed in [22–27]. The references have investigated possible relationships between the bivariate Hurst exponent $H_{xy}$ and an average of the separate Hurst exponents $(H_x + H_y)/2$. In this paper, we investigate possible relationships between the bivariate $q$-order Hurst exponent $H_{xy}(q)$ and an average of the separate $q$-order Hurst exponents $(H_x(q) + H_y(q))/2$ by using different multifractal cross-correlation methods. Many numerical experiments are carried out to study the performances of different methods such as MFXPF, MFXDFA and MFXDMA to well-established mathematical models (binomial multiplicative cascades (BMC) and Cantor sets), which have known analytical expressions. We also apply these methods to the IPIX radar sea clutter data and unveil intriguing multifractality in the cross correlations of the sea clutters with and without target. The results provide us a guideline for the target detection within sea clutter.

This paper is organized as follows. Section 2 describes a unified framework of the MFXPF, MFXDFA and MFXDMA algorithms. Section 3 performs extensive numerical experiments by using multifractal time series with known analytical expressions (BMC and Cantor sets) to investigate the performances of the algorithms. In Section 4, we apply the algorithms to the IPIX radar sea clutter data. We summarize our main findings in Section 5.

## 2. Algorithm models

### 2.1. The multifractal cross-correlation analysis based on the partition function approach

The MFXPF $(q)$ method reduces to the QMSPF method [7] when $\{X(i)\} = \{Y(i)\}$. It is the simplest type of multifractal analysis based upon the standard partition function multifractal formalism, which has been developed for the multifractal characterization of normalized, stationary measures. Unfortunately, this standard formalism does not give correct results for nonstationary time series that are affected by trends or that cannot be normalized [28]. So, to make sure that the input time series can be normalized, there are two requirements of the input data. Firstly, there are no trends in the time series. Secondly, every value in the time series is larger than zero, or most values of the time series are larger than zero and a few values are equal to zero. The MFXPF $(q)$ method, which is written as "MFXPF" in the following sections, can be summarized as follows:

**Step 1**. Consider two time series $\{X(i)\}$ and $\{Y(i)\}$ of the same length $N = 2^n$, $X(i) \geq 0$, $Y(i) \geq 0$, $i = 1, 2, 3, \ldots, N$. Form two new series $\{x(i)\}$ and $\{y(i)\}$, $i = 1, 2, 3, \ldots, N$, where $x_k = X_k / \sum_{j=1}^{N} X_j$, $x_k \geq 0$, $\sum x_k = 1$, $y_k = Y_k / \sum_{j=1}^{N} Y_j$, $y_k \geq 0$, $\sum y_k = 1$.

**Step 2**. Divide the two new series into $N_s = \lfloor N/s \rfloor$ non-overlapping segments of the same size $s$, $s = 1, 2, 2^2, \ldots, 2^n$, we can obtain the box probability of each segment

$$p_{xv}(s) = \sum_{k=(v-1)*s+1}^{vs} x_k, \quad v = 1, 2, \ldots, N_s, \tag{1a}$$

$$p_{yv}(s) = \sum_{k=(v-1)*s+1}^{vs} x_k, \quad v = 1, 2, \ldots, N_s. \tag{1b}$$

**Step 3**. The $q$-order moment structure partition function can be obtained for variant $s$ as follows:

$$Z_q(s) = \sum_{v=1}^{N_s} |p_{xv}(s)p_{yv}(s)|^{q/2}. \tag{2}$$