Computers and Structures 146 (2015) 20–31

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Non-parametric shape optimization method for natural vibration design of stiffened shells

Yang Liu [a,*], Masatoshi Shimoda [b]

[a] Department of Mechanical Engineering, Osaka University, 2-1, Yamadaoka, Suita, Osaka 567-0047, Japan
[b] Department of Advanced Science and Technology, Toyota Technological Institute, 2-12-1 Hisakata, Tenpaku-ku, Nagoya 468-8511, Japan

ARTICLE INFO

Article history:
Received 8 February 2014
Accepted 21 August 2014
Available online 1 October 2014

Keywords:
Natural vibration
Eigenvalue maximization
Volume minimization
Shape optimization
Non-parametric
Stiffened shells

ABSTRACT

In this paper, we newly present an effective shape optimization method for natural vibration design of stiffened thin-walled or shell structures. Both the stiffeners and their basic structures are optimized by solving two kinds of optimization problems. The first is a specified eigenvalue maximization problem subject to a volume constraint, and the second is its reciprocal volume minimization problem subject to a specified eigenvalue constraint. The boundary shapes of a thin-walled structure are determined under the condition where the stiffeners and the basic structure are movable in the in-plane direction to their surface. Both problems are formulated as distributed-parameter shape optimization problems, and the shape gradient functions are derived using the material derivative method and the adjoint variable method. The optimal free-boundary shapes are determined by applying the derived shape gradient function to the H^1 gradient method for shells, which is a parameter-free shape optimization method proposed by one of the authors. Several design examples are presented to validate the proposed method and demonstrate its practical usages.

1. Introduction

Thin-walled or shell structures are widely used as basic structural components in various industrial products such as car bodies, aircraft fuselages, pressure vessels as well as in bridges and buildings. They are commonly stiffened by stiffeners to improve the bending rigidity of the basic structures. With recent enhancements of high speed, high function and substantial weight reduction of thin-walled structures, the vibration design in consideration of the dynamic characteristics has become more important than ever. The natural frequencies (i.e., vibration eigenvalues) usually represent the dynamic characteristics of structures, especially the lower order natural frequencies are considered as an evaluation measure of the dynamic stability. The dynamic response of structures can be reduced by increasing their lower order natural frequencies [1,2]. Moreover, the reduction of the dynamic response of a structure generally leads to the minimum weight for the structure design [3].

In terms of the optimum design of stiffened thin-walled structures under either static or dynamic loading conditions, investigations have been extensively carried out to achieve better static or dynamic performance as well as a lighter weight. Most of the investigations focused on determining the best layout configurations for stiffeners on thin-walled structures. For example, Cheng and Olhoff [4] reported a method of generating the optimal stiffener layout pattern for maximizing the integral stiffness of a solid elastic plate by using the plate thickness function as the design variable. Luo and Gea [5,6] used a systematic topology optimization based approach to design the optimal location and orientation of stiffeners for static and interior sound reduction problems. Liu et al. [7] studied the eigenvalue sensitivity with respect to the location of stiffeners for a stiffened plate. Ding and Yamazaki [8] introduced a growing and branch tree model to generate stiffener layout patterns on plate structures for vibration design problems. Bojczuk and Szteleblak [9] showed an application of a method based on sensitivity analysis combined with an adjoint method to the optimization of 2D structures with respect to the deployment of stiffeners. Many investigations have also dealt with size design optimization of stiffener geometrical properties, such as their number [10,11], thickness [12], cross-section dimensions [13,14], and spacing [15]. On the other hand, few studies contribute to the shape optimization of the stiffeners and their basic structure, though the optimum shape design can greatly influence the static and dynamic characteristics as well as weight [16,17]. As one of the few studies, the authors recently developed a parameter-free shape optimization method to deal with the shape optimum design of stiffeners on thin-walled structures [18]. In this

* Corresponding author. Tel.: +81 6 68794121; fax: +81 6 68794121.
E-mail addresses: liuyang60212@gmail.com, ryu@comec.mech.eng.osaka-u.ac.jp (Y. Liu), shimoda@toyota-ti.ac.jp (M. Shimoda).

http://dx.doi.org/10.1016/j.compstruc.2014.08.003
0045-7949/

method, we employed the adjoint variable method and the shape updating approach by traction force to solve difficulties of the large-scale design variables problem and the jagged boundary problem [19] in the parameter-free shape optimization. Bletzinger [20] proposed the sensitivity filtering technique to solve these difficulties in their parameter-free method. To the best of our knowledge, no other group has applied the parameter-free method to the stiffener design problem. However, only the compliance minimization problem was solved simply as a self-adjoint problem in our previous work [18], in which the basic structure was assumed not to be varied. It is not available for the vibration design in consideration of the dynamic characteristics, which is known as a more complicated design problem.

For the natural vibration problem of stiffened shells, this paper newly presents a parameter-free shape optimization of both the stiffeners and their basic structures on stiffened shells. Two kinds of natural vibration design problems are treated as parameter-free shape optimization problems. One is a specified eigenvalue maximization problem subject to a volume constraint, and the other is its reciprocal volume minimization problem subject to a specified eigenvalue constraint. We formulate the two design problems in the continuous system, or in the function space, which enables our method to create the optimal shapes without any shape parameterization and discretization in advance. In other words, the shape obtained is not influenced by the parameterization and the discretization. Sensitivity functions (i.e., shape gradient functions) for both stiffeners and the basic structure are theoretically derived using the material derivative method and the adjoint variable method. The direct derivatives of the element stiffness matrix are not required in the sensitivity calculation. Therefore, it can be easily implemented in combination with a commercial FEM code and the shape optimization of practical shell structures is easily computable even if it is complicated. After that, the negative shape gradient function derived is applied as a distributed force to free boundaries of the stiffeners and the basic structure to vary the shapes. This approach makes it possible both to reduce the objective functional and to maintain the mesh regularization simultaneously. Moreover, the target vibration mode is considered as the one receiving the most attention or being disadvantageous in the practical design. To eliminate difficulties caused by repeated eigenvalues, i.e., mode switching or frequency crossing during optimization [21], the Modal Assurance Criterion (MAC) [22] is adopted to track the specified natural mode through changes in the eigenvalue maximization or eigenvalue constraint problem.

In the following section, the governing equation of the natural vibration of a shell will be described. Next, the formulation of design problems and the derivation of each shape gradient function will be presented in Section 3. After explaining the details of the optimization method in Section 4, the validity and practical utility of this method will be verified through several design examples in Section 5.

2. Variational equation for natural vibration of shell modeled by infinitesimal flat plates

As shown in Fig. 1 and Eqs. (1)–(3) basic shell structure or stiffener with an initial bounded domain $\Omega \subset \mathbb{R}^3$ is defined by the mid-area A and the domain of thickness direction $(-h/2, h/2)$, and the side surface S is defined by the boundary ∂A of the mid-area A.

$$\Omega = \{(x_1, x_2, x_3) \in \mathbb{R}^3 | (x_1, x_2) \in A \subset \mathbb{R}^2, \quad x_3 \in (-h/2, h/2)\}, \tag{1}$$

$$\Omega = A \times (-h/2, h/2), \tag{2}$$

$$S = \partial A \times (-h/2, h/2). \tag{3}$$

In the structural analysis of a shell with arbitrary geometry, a practical approach is to model the shell by a set of infinitesimal flat plates, that is not only for simplicity but also frequently performs quite well in curved shell applications [23]. As shown in Fig. 1, each flat plate dA has a local coordinate system (i.e., plate coordinate system) that is fixed with respect to its geometry and independent of the unique coordinate system used at all points on the shell (i.e., global coordinate system). Both coordinate systems are constructed as a Cartesian coordinate system. The transformation between degrees of freedom in the local coordinates, u^l, and in the global coordinate, u^g, is calculated in Eq. (4).

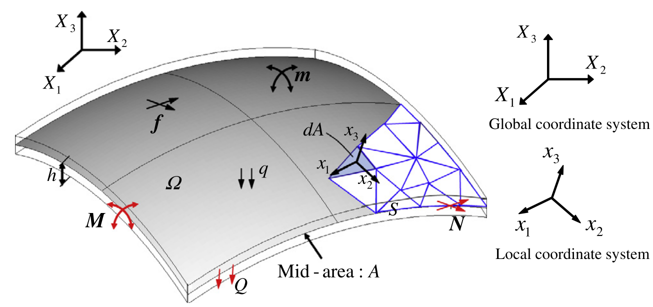

Fig. 1. Shell geometry assembled by infinitesimal flat plates.

$$u_j^l = T_{ij}^{gl} u_i^g, \tag{4}$$

where T^{gl} indicates the global–local transformation tensor. The displacement expressed by the local coordinates $u^l = \{u_i^l\}_{i=1,2,3}$ is considered by dividing it into the displacement in the in-plane direction u_α and the displacement in the out-of-plane direction u_3^l. In this paper, the subscripts of the Greek letters are expressed as $\alpha, \beta, \gamma, \delta = 1, 2$, the tensor subscript notation uses Einstein's summation convention and a partial differential notation for the spatial coordinates $(\cdot)_{,i} = \partial(\cdot)/\partial x_i$.

The Mindlin–Reissner plate theory is applied for each flat plate dA and a plate bending stiffness and membrane stiffness are combined in its local coordinate system [24]. In the local coordinate system of dA shown in Fig. 2, the membrane action is expressed by the in-plane displacement $u_{0\alpha}$ of the mid-area, and the bending action is expressed by the out-of-plane displacement w, and the rotational angle θ_α. The Mindlin–Reissner plate theory posits the following conditions with respect to the displacement of a general point on dA in its local coordinate system.

$$u_\alpha^l(x_1, x_2, x_3) \equiv u_{0\alpha}(x_1, x_2) - x_3\theta_\alpha(x_1, x_2), \tag{5}$$

$$u_3^l(x_1, x_2, x_3) \equiv w(x_1, x_2), \tag{6}$$

When assembling the flat plates to model the curved shell, the bending and membrane stiffness are coupled only on the interelement boundaries due to differences between adjacent plate orientations. Since the shell dealt with in our manuscript is assumed to be assembled by infinitesimal flats, the differences should be extremely small and the coupling effects can be neglected. Moreover, it should be remarked that the formulation of a shell modeled as an assembly of flat plates requires the handling of different coordinate systems, and the bending stiffness and the membrane stiffness as well as the force vector should be transformed from local coordinates to the global coordinate. However, for the sake of brevity, this

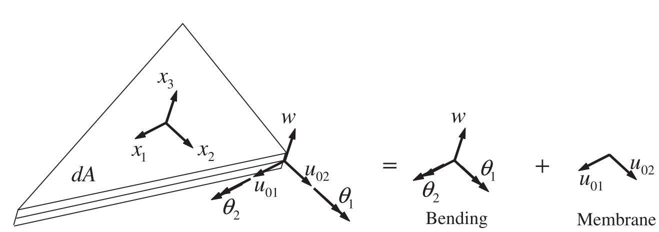

Fig. 2. Formulation of a infinitesimal flat plate dA in its local coordinate system.