Computers and Structures 144 (2014) 1–11

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

A non-parametric solution to shape identification problem of free-form shells for desired deformation mode

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ARTICLE INFO

Article history:
Received 3 December 2013
Accepted 29 July 2014
Available online 27 August 2014

Keywords:
Shape identification
Stiffness control
Curvature distribution
Free-form shells
Non-parametric
Traction method

ABSTRACT

A shape identification method of free-form shells is presented for controlling the static deformation mode to the desired one. This problem is formulated as a parameter-free shape optimization problem, in which a squared displacements error norm on the prescribed region is employed as an objective functional. The shape sensitivity, called shape gradient function, is theoretically derived using the adjoint variable method and the formula of the material derivative, and then applied to a gradient method with Laplacian smoother for shells to determine the smooth optimal shape. Several calculated examples are presented to verify the validity and practical utility of the proposed method.

1. Introduction

Shell structures are widely used as basic components in various kinds of structures in civil, architectural, mechanical, aeronautical, and marine engineering. Since shapes or curvature distributions of shells greatly influence its mechanical properties and weight, it is strongly required to find their optimal shapes so as to satisfy various mechanical characteristics, functions and artistic impression if required. Almost all the literatures regarding shells [1,2] are to obtain a maximization of the mechanical characteristics, or a minimum weight. However, in the practical design of shell structures, a geometrical shape constraint of controlling the deformation mode to be a desired one is also necessary to be considered for attaining the precision requirement or achieving an imposed function on the structure. One of the optimum design under the geometrical shape constraint is homology design. The concept of homology design was proposed by Hoerner in the design of large radio telescopes [3], where the deformation of a structure was defined as homologous if a given geometrical relation holds for a given number of structural points before, during, and after the deformation. Later, Yoshikawa et al. proposed a formulation based on the finite element sensitivity analysis for homology design of frame structures [4,5]. Shimoda et al. presented shape and topology optimization methods of continua for homologous deformation using the traction method [6,7]. Lee et al. developed a truss optimization method using equality equations to include homology constraints under multiple loading conditions as well as single loading conditions [8,9]. Shin et al. employed the homologous design to a nuclear fuel spacer grid spring, which supports the fuel rods in a nuclear fuel system, to reduce the fretting wear while maintaining the functions of the spring [10]. Another approach for controlling the deformation mode is based on the optimum design of compliant mechanism. In designing compliant mechanism, the displacements of the loading area (i.e., input displacements) and the specified region (i.e., output displacements) are controlled to be a desired value in the specified direction, in order to achieve a flexible structure with mechanical function. Although the objective of the compliant mechanism design is similar to the homology design, the design methods for compliant mechanisms are mainly based on topology optimization. Design of compliant mechanism using topology optimization was firstly introduced by Ananthasuresh et al. in 1994 [11]. Subsequently, researches on compliant mechanism with topology optimization were extended to various optimization formulations [12–14]. However, current researches on the homology design and the compliant mechanism are limited to 2D and 3D continua, and the optimum design of free-form shells for achieving a desired deformation mode has not been discussed.

Focusing on shape optimization method of the shell structure, it is categorized into parametric and non-parametric methods in

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http://dx.doi.org/10.1016/j.compstruc.2014.07.020
0045-7949/

terms of design variables. Most of previously proposed shape optimization methods for shells are parametric methods [15–17]. The parametric methods are effective to reduce the design variables, but require a shape parameterization for complicated structure in advance and the obtained shapes are strongly dominated by the parameterization process. On the other hand, in the non-parametric methods, i.e., node-based methods, all of the nodal coordinates can be taken as design variables. Due to the result is not restricted to the parametrization, the non-parametric methods give more freedom to the optimization process and an optimal shape obtained is near a result of natural choice. Difficulties of the non-parametric methods are that it has to deal with a large number of design variables and to overcome the jagged boundary problems, which were pointed out in one of the earliest works on shape optimization by Braibant and Fleury [18]. The adjoint variable method is commonly used in the sensitivity analysis to resolve the former large-scale problem, and filtering techniques have been developed as smoothing solutions to the latter jagged shape problem. Among those techniques, Bletzinger et al. proposed a mesh independent regularization method based on sensitivity filtering in the shape updating process [19–22]; Le et al. introduced an shape filtering approach by filtering actual values of the design variables [23]; Hojjat et al. reported a vertex morphing method to perform the out-of-plane filtering and in-plane mesh regularization operators simultaneously [24]. There are also filtering techniques as developed for CFD problems [25,26]. Alternatively, Shimoda et al. proposed a parameter-free optimization method for shells based on the traction method [27,28], which is a type of gradient method in the Hilbert space [29,30]. In this method, the adjoint variable method was also employed and fictitious forces were used to vary the surface shape, and to reduce the objective functional while maintaining the mesh regularity [31]. In our previous work, this method was expanded to deal with a parameter-free shape optimization of stiffeners on thin-walled structures [32].

In this study, we develop this method to solve a shape identification problem of linear elastic free-form shells for the purpose of achieving a desired deformation mode under external forces. Controlling the displacement distribution to a given desired one can contribute to solving compliant design problems of thin-walled structures, which means that the solution described here can impart a function to structures by simply changing their shapes. Moreover, a stiffness control problem can be achieved by defining the displacements of the loading points to the desired values. In this paper, firstly, the shape identification problem is formulated as a parameter-free shape optimization problem, in which the desired deformation mode is identified by introducing a squared error displacements norm of a deformed shape on its prescribed surface as the objective functional. Subsequently, sensitivity function (i.e., shape gradient function) for this problem is theoretically derived using the material derivative method and the adjoint variable method. After that, the negative shape gradient function derived is applied in the normal direction to the pseudo-elastic shell as a fictitious distributed traction force to vary the shapes. This approach makes it possible both to reduce the squared error displacements norm and to maintain the mesh regularization, simultaneously. With the proposing method, an optimum shell structure with a smooth free-form surface and a desired deformation mode can be obtained without any shape parameterization.

2. The weak-formed governing equation for a shell modelled by infinitesimal plates

As shown in Fig. 1, consider a shell consisting of an initial bounded domain $\Omega \subset \mathbb{R}^3$ with boundary of $\partial\Omega$, mid-area A with the boundary of ∂A, side surface S and plate thickness h. An in-plane load $\boldsymbol{f} = \{f_\alpha\}_{\alpha=1,2}$, an out-of-plane moment $\boldsymbol{m} = \{m_\alpha\}_{\alpha=1,2}$ and an out-of-plane load q per unit area are applied on A, and an in-plane load $\boldsymbol{N} = \{N_\alpha\}_{\alpha=1,2}$, a bending moment $\boldsymbol{M} = \{M_\alpha\}_{\alpha=1,2}$ and a shearing force Q per unit length are applied on ∂A. As a practical analysis approach to free-form shell with arbitrary geometry, a general linear elastic shell is modelled by a set of infinitesimal flat plates, that is not only for simplicity but also frequently performs quite well in curved shell applications [33]. Each flat plate element has a local coordinate system (i.e., element coordinate system) that is fixed with respect to the element's geometry and independent of the unique coordinate system used at all nodes (i.e., global coordinate system). The transformation between nodal degrees of freedom in the local coordinates, $\boldsymbol{u}^e$, and nodal degrees of freedom in the global coordinate, $\boldsymbol{u}^g$, is calculated in Eq. (1).

$$u_j^e = T_{ij}^{ge} u_i^g \tag{1}$$

where $\boldsymbol{T}^{ge}$ indicates the global–local transformation matrix.

The plate bending theory used in this paper is based on the Reissner–Mindlin theory [34], in which membrane action and bending action are combined in the local coordinate system as shown in Fig. 2. Since the plate shells are assumed to be infinitesimal, coupling effects of the membrane component and the bending component should be extremely small and can be neglected for simplicity [33]. The details of assembling the bending stiffness and the membrane stiffness as well as the force vector to the global coordinate system are introduced in the reference [35].

The Reissner–Mindlin plate theory assumes the following conditions with respect to displacement in the local coordinate system of each plate shell.

$$u_\alpha^e(x_1, x_2, x_3) \equiv u_{0\alpha}(x_1, x_2) - x_3\theta_\alpha(x_1, x_2), \tag{2}$$

$$u_3^e(x_1, x_2, x_3) \equiv w(x_1, x_2), \tag{3}$$

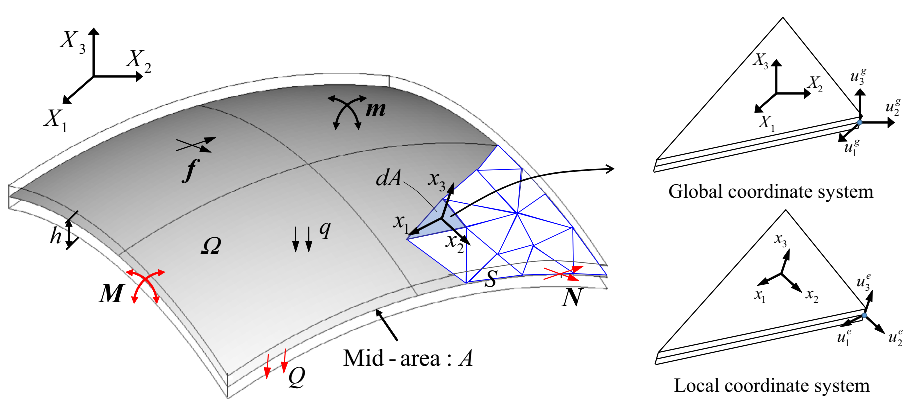

Fig. 1. Shell geometry assembled by infinitesimal flats.