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Structural optimisation based on the boundary element and level set methods

B. Ullah *, J. Trevelyan, P.C. Matthews

School of Engineering and Computing Sciences, Durham University, South Road, Durham DH1 3LE, UK

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ABSTRACT

A new method of structural topology optimisation is proposed in which an evolutionary approach is used with boundary element and level set methods. During the optimisation iterations, the proposed method automatically introduces internal cavities and does not rely on an initial guess topology with pre-existing holes. The zero level set contours describing both the external geometry and the internal cavities are converted to non-uniform rational B-splines (NURBS) for smooth boundary element meshing at each iteration. The optimal geometries generated by the proposed method for two-dimensional cases closely resemble to those available in the literature for a range of benchmark examples in the field of topology optimisation.

1. Introduction

Structural engineers worldwide are driven by the search for a design that is in some sense optimal, making the most efficient use of materials. In order to support this search, an extensive body of literature has appeared over the last decades describing various numerical techniques to generate structures that are optimal in terms of quantities such as weight, cost and stiffness. Most schemes in the literature make use of the finite element method (FEM) to perform the structural analysis that guides the optimisation process. Methods that have enjoyed enduring popularity include the homogenisation method of Bendsøe and Kikuchi [1], based on varying the material porosity. This was enhanced to improve the stability for practical usage with the development of the SIMP method by Rozvany et al. [2].

The most challenging structural optimisation problems are those of topology optimisation, which remains an active research area. Eschenauer et al. [3] introduced the bubble method, which is based on the insertion of new holes in the structure and the subsequent use of a shape optimisation method to determine their optimal size and shape. The concept of adaptive topology optimisation, developed by Maute and Ramm [4], is based on the smoothness of the effective design space with a cubic or Bézier spline approximation based on the density distributions. This procedure not only reduces the number of design variables but also provides smooth geometry. Papalambros and Chirehdast [5] presented a three phase, homogenisation-based approach to integrated structural optimisation with CAD.

The inspiration from nature, i.e. how structures such as bones, trees and shells achieve their optimum over a period of time under specific environmental conditions, led to the development of the evolutionary structural optimisation (ESO) method. The simple evolutionary method presented by Xie and Steven [6] progressively removes material (i.e. finite elements) from low stress regions based on some rejection criteria. Similarly in Bi-directional ESO [7,8], material removal is accompanied by material addition in highly stressed regions. Garcia and Steven [9] introduced the concept of Fixed Grid (FG) FE analysis to simplify the meshing in order to enhance computational efficiency in problems where geometry changes with time. This is attractive from the point of view of efficiency, but the accuracy of stresses in elements intersecting the problem boundaries may become compromised. Dunning et al. [10] have used FG-FE simulations to drive a sensitivity based scheme for topology optimisation in the presence of uncertainty in the loading.

There has been some controversy over the last decade over the validity of ESO as an optimisation approach when the removal and addition of material is provoked by local stress values, in contrast with the use of design sensitivities related to an objective function. In spite of this, stress based ESO schemes have remained popular on account of their simplicity and extensive empirical evidence of the fact that their optimal solutions closely resemble those derived by more rigorous descent methods (e.g. Li et al. [11]).

While finite elements have been a popular method, they have some shortcomings when used as the analysis engine for optimisation methods. Haftka and Grandhi [12] highlighted the principal

* Corresponding author. Tel.: +44 1913342380.
E-mail addresses: baseer.ullah@durham.ac.uk (B. Ullah), jon.trevelyan@durham.ac.uk (J. Trevelyan), p.c.matthews@durham.ac.uk (P.C. Matthews).

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issue in shape optimisation, that it is difficult to ensure the accuracy of the analysis for a continuously changing finite element model; the change in the shape of a structure distorts the shape of the finite elements, with consequent deterioration in the accuracy of the stress solution. For these reasons it has been popular to use fixed grid FE approaches [9] to reduce distortion. However, poorly shaped elements still remain. The requirement of a smooth optimal geometry further increases the computational cost due to high mesh refinement at the boundaries. This leads us to propose the boundary element method (BEM) as an appealing alternative. The BEM is a well-established alternative to the FEM in structural analysis, and is attractive because it requires discretisation only at the structural boundary. This reduction of problem dimensionality considerably simplifies the re-meshing task, which can be performed efficiently and robustly. Thus, its rapid and robust re-meshing and accurate boundary stress solutions make the BEM a natural choice in the field of shape and topology optimisation.

While the BEM has been exploited for structural optimisation in earlier works [13–15] it is topology optimisation on which this paper focusses. Cervera and Trevelyan [16,17] used BEM for topology optimisation of two and three dimensional problems. In their ESO approach the moving geometry of the structure was represented by NURBS [18] explicitly, the spline control points being moved in response to local stress values. The boundary element based topological derivatives concept was used for the first time by José Marczak [19] for the topology optimisation of thermally conducting solids. The proposed formulation was based on the concept of introducing an iterative material removal procedure in a BEM framework. Carretero Neches and Cisilino [20] presented topology optimisation of 2D elastic structures using the BEM with linear elements, inserting small holes in the model around internal points with the lowest values of the topological derivative. Bertsch et al. [21] presented three dimensional elastic topology optimisation in a BEM framework with the topological shape sensitivity method for the direct calculation of topological derivatives from stress fields.

The level set method (LSM) presented by Osher and Sethian [22] has emerged as a powerful tool for describing the evolution of moving boundaries. It is particularly powerful in its ability to deal with complex merging and separation of different boundaries. There have been several examples in the literature of researchers exploiting this in topology optimisation, firstly by Sethian and Wiegmann [23] and later Wang et al. [24]. Numerical shape derivatives were used by Allaire et al. [25] for structural optimisation in 2D and 3D with both linear and nonlinear elasticity models. However, their approach is restrictive in that no new holes can be nucleated in 2D structural optimisation; moreover, the optimum solution is highly dependent on the initially guessed topology. Allaire and Jouve [26] combined the shape derivatives with topological derivatives to present a level set based optimisation method capable of automatic hole insertion. The proposed approach was shown to be independent from local minima but the implementation of topological derivatives is very difficult in numerical practice [27,28] because, the hole size is dependent on a single mesh cell which cannot be infinitesimally small as proposed in the method [26]. In addition, the resulting optimal structure depends on the values of various parameters which can affect the stability of the optimisation process [29]. Other examples of LSM combination with FEM-based structural optimisation schemes can be found in [29–31].

The use of BEM with the level set method in structural optimisation was first used by Abe et al. [32]. During each optimisation iteration the evolving structural boundary is re-constructed from the zero level set contours, which consists of line segments joining the zero level set intersection points. The resulting non-smooth geometry is then meshed with linear boundary elements to perform the sensitivity analysis for the next iteration. The non smooth geometry and the linear boundary elements greatly reduce the accuracy of the expensive sensitivity calculations, and hence the method requires a large number of iterations to achieve convergence. In addition the use of sensitivity analysis restricts the nucleation of new holes and makes this method highly dependent on the initially guessed topology.

This paper presents an initial study of the integration of BEM, evolutionary optimisation approach, LSM and NURBS for 2D structural optimisation problems. The proposed method uses the 2D version of the BEM analysis software Concept Analyst (**CA**) [33]. The approach overcomes many of the shortcomings of earlier works; boundaries remain smooth throughout, and holes are inserted automatically revealing the final topology from a simple starting geometry. This paper is organised as follows. The basic details of LSM are introduced in Section 2, and the BEM is developed in Section 3. In Section 4 we present the details of the optimisation algorithm and its implementation. The results obtained from the proposed algorithm are presented and discussed in Section 5, and the paper closes with some concluding remarks in Section 6.

2. Level set method

The LSM is an efficient numerical technique developed by Osher and Sethian [22] for the tracking of propagating interfaces. The wide variety of applications in which LSM is successfully implemented include computer vision, medical scans, seismic analysis, fluid flow, structural optimisation and optimal control. The propagation of the structure boundary during the optimisation can be linked with the evolution of the function ϕ as an initial value problem. This means that the position of the structure boundary at any time t is given by the zero level set function ϕ. Therefore the evolution equation of the LSM given in [22] is

$$\frac{\partial \phi}{\partial t} + F|\nabla \phi| = 0 \tag{1}$$

where F is the velocity in the normal direction and t is the virtual time.

In the implicit representation the connectivity of the discretisation does not need to be determined explicitly. This is one of the most interesting features of the implicit geometric representation, in that merging and breaking of curves in 2D and surfaces in 3D can be handled automatically. Thus in this work the holes appear, merge and vanish automatically. It is worth mentioning that, although we are not solving time-dependent problems, the LSM uses virtual time to describe the advancing front.

The implicit method uses the Eulerian approach to represent an evolving geometry. In 2D this method works on an underlying fixed Cartesian grid. The geometry of the structure to be optimised is embedded as the zero level set of a higher dimensional function ϕ. The value of ϕ is the distance of a particular grid point from the boundary with a sign to indicate points either inside or outside of the boundary. We define Ω^- as the region contained within the boundary, Ω^+ as the union of the regions inside holes and the region of the design domain outside the boundary, and the contour $\partial\Omega$ as the interface between the non-overlapping regions Ω^- and Ω^+. These definitions are expressed as follows and shown in Fig. 1.

$$\phi(\vec{x}) \begin{cases} < 0 & \vec{x} \in \Omega^- \\ = 0 & \vec{x} \in \partial\Omega \\ > 0 & \vec{x} \in \Omega^+ \end{cases} \tag{2}$$

3. Boundary element method

The Boundary Element Method (BEM) is a standard technique for computational solution of partial differential equations. There