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Quasi-hinge beam element implemented within the hybrid force-based method

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ABSTRACT

This paper describes a new force-based hinge element implemented in the framework of the Large Increment Method (LIM). The element can be of arbitrary cross section and is capable of including inelastic behaviour close to structural hinges. The element formulation can accommodate elasto-plastic strain hardening material behaviour. The solution procedure involves the analysis of elastic and inelastic deformations separately facilitated by splitting of the element length into elastic and inelastic zones. Deformation is calculated by considering inelastic behaviour in the element volume close to both ends of the structural member using an optimum number of integration points in order to achieve good accuracy while maintaining computational efficiency. The predictions of both conventional- and quasi-hinge elements are compared against predictions from Abaqus™. Predictions of the quasi-hinge element show significant improvements over the conventional-hinge method and are shown to converge on the Abaqus™ prediction as the number of monitoring sections in the element is increased.

1. Introduction

The possible occurrence of inelastic deformations when a structure experiences either earthquake or blast loading, can be a significant concern when designing structures. Reducing the computational time associated with modelling and analysis of inelastic structures is an important goal in structural engineering. Generally, inelastic behaviour in frame structures can be studied using two main approaches (i) the Distributed Inelastic Method (DIM), which can be further subdivided into techniques using either customised fibre elements or, more commonly, using continuum elements and (ii) the Concentrated Inelastic Method (CIM). In the fibre-based DIM, each structural member in the frame is modelled by numerous fibres along the length and over the cross section of each element. The fibre-based DIM enables both stress and strain to be determined along the length and through the thickness of the structural member during an analysis. This permits calculation of the gradual spread of inelastic behaviour over the member cross-section and length as deformation proceeds. The fibre-based DIM can provide an accurate solution, enable tracking of phenomena such as cracking and residual stress while being much less demanding in terms of computational resource than a typical full general DIM based on continuum elements. Nevertheless, the computational cost of even the fibre-based DIM can still be prohibitive for certain problems. In such cases, a CIM can provide an alternative and faster method when inelastic behaviour is considered. Using this method, a single element with multiple integration points is used to model each structural member.

When a frame structure is subjected to lateral forces above its yielding load, most of the inelastic material response is often observed to be concentrated towards the ends of the frame's structural members. This observation has prompted the development of the CIM (also known as the plastic hinge or lumped inelastic method). The latter is a computationally efficient method to represent inelasticity in structural frame members. Along the majority of its length a beam usually remains elastic; it is usually only towards the hinges that the elastic capacity of the beam's section is passed. In the conventional implementation of this method, a zero-length hinge is assumed while the rest of the element's behaviour remains elastic [1]. This implies that, as with the fibre-based DIM, just one beam-column element per structural member can capture the inelastic behaviour of the entire structure. This is in contrast with the continuum-based DIM, which involves numerous distinct elements in modelling each structural member. However, a limitation of the conventional CIM is that inelastic behaviour can only be considered at the very ends of a structural beam member. The method is also incapable of including gradual plasticisation of the hinges, i.e. the gradual increase in length of the plastic zone near the hinges. The resulting element accuracy is consequently affected.

The displacement-based solution strategy involves minimising the strain energy in a structure. This means the final solution is either equal to, or very slightly higher than the minimum possible

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Glossary of symbols

The following convention is used in this paper: matrices and second order tensors are written in bold using upper-case symbols, vectors are written using bold lower-case symbols while scalar quantities are written using regular upper and lower case symbols. For convenience a glossary of symbols is given below:

Variables	*Definition*
$\boldsymbol{\Omega}$	body domain
$\boldsymbol{\Omega}^e$	elastic body domain
$\boldsymbol{\Omega}^p$	plastic body domain
Δ	deformation vector
ε	strain tensor
σ	stress tensor
ε_{ij}	strain vector components
σ_{ij}	stress vector components
ε_N	axial strain
θ_i, θ_j	rotation of the ends of the beam element
θ^e	elastic contribution towards rotation at ends of the beam element
θ^p	plastic contribution towards rotation at ends of the beam element
θ_y	section rotation with respect to the y axis
ϕ_N, ϕ_i, ϕ_j	stiffness reduction factors
μ	section ratio
A	element cross section
A^e	elastic cross section
A^p	plastic cross section
b	width of cross section
b_i	body force vector components
$\boldsymbol{b}$	body force vector
$\boldsymbol{B}$	strain–displacement matrix
$\mathcal{B}$	unbalanced load definer matrix
$\boldsymbol{C}$	equilibrium matrix
$\boldsymbol{C_r}^{-1}$	right side inverse matrix
$\boldsymbol{d}$	nodal displacement vector
$\boldsymbol{D_m}$	material constitutive matrix
$\mathcal{D}_{\boldsymbol{m}}$	section constitutive inverse matrix
E	elastic modulus
E_t	inelastic modulus
$\bar{f}_i$	nodal force component
f_i	elemental force component
f_s	section shape function
$\boldsymbol{f}_{Se}$	section force vector
f_i^e	elastic flexibility matrix components
δf_i^p	inelastic flexibility matrix components
$\bar{\boldsymbol{f}}$	nodal force vector
$\boldsymbol{f}$	elemental force vector
$\boldsymbol{\delta f}$	change in elemental force vector (unbalanced load vector)
$\boldsymbol{F_{Se}}^{\boldsymbol{e}}$	elastic section flexibility matrix
$\boldsymbol{F_{Se}}^{\boldsymbol{p}}$	inelastic section flexibility matrix
$\boldsymbol{F}$	flexibility matrix
$\boldsymbol{F^e}$	elastic flexibility matrix
$\boldsymbol{F^p}$	inelastic flexibility matrix
h	element section height
h_0	conjugate gradient modifier
H_i	height of story i
$\boldsymbol{I}$	identity matrix
$\boldsymbol{K}$	stiffness matrix
k_0, k_1, k_2, k_3	stiffness parameters
L	element length
L^e	elastic element length
L^p	inelastic element length
L_i^p, L_j^p	inelastic length next to end i and j
M_y	moment about y axis
M_p	plastic moment capacity
M_r^p	reduced plastic moment capacity
M_i, M_j	moments at both i, j ends
n_u	nodal degree of freedom number
n_f	elemental degree of freedom number
N	axial force
N_y	section axial strength capacity
$\boldsymbol{N}$	shape function matrix
$\boldsymbol{p}$	external load vector
$\boldsymbol{Q}(\boldsymbol{x})$	section force definition matrix
S_c	shape calibration factor
$\boldsymbol{s}$	search direction vector
s	boundary surface
s^e	elastic boundary surface
s^p	plastic boundary surface
t_i	surface traction force components
$\boldsymbol{t}$	surface traction force vector
u_i	displacement vector components
$\boldsymbol{u}$	deformation vector
$\boldsymbol{u}^e$	elastic deformation vector contribution
$\boldsymbol{u}_i^p$, $\boldsymbol{u}_j^p$	inelastic deformation vector contributions at either end of element
x_i	local coordinate system
x	coordinate aligned with beam length
Δx	displacement shift in x direction
z	distance from the neutral plane
Δz	displacement shift in z direction
Z	force shape function
A^*, B^*, C^*, a_1, a_2, b_1, b_2, c_1	dummy variables

theoretical energy for the structure. Consequently, the final numerical prediction is usually a very slight overestimation compared to both the theoretical and also the actual stiffness of the structure [2]. This is the case for all elements implemented using a displacement based solution strategy, including hinge elements [3]. As a consequence two methods of improving the conventional hinge element predictions have been proposed; the first is the 'refined hinge' method which, using a displacement-based approach involves the use of stiffness reduction factors to modify the original elastic stiffness matrix, the second is the 'quasi-hinge' method which involves the implementation of a non-zero hinge length. The results of both methods are closer to the exact answer, compared to the conventional zero-length hinge method. These hinge-based methods can be much more computationally efficient than a conventional DIM [3] while still providing results of satisfactory accuracy for most practical purposes [4–6].

In much of the previous research appearing in the literature, hinge elements based on the CIM have been developed and implemented within the framework of relatively mature displacement-based finite element solution strategies. An issue with these displacement-based solution techniques is error accumulation caused by linearisation; after each load increment or step, an iteration procedure, involving a linearisation process, is conducted to minimise any residual error in the solution. This error increases slightly following each step due to the accumulation of small residual errors that remain following each step [7]. Normally, the convergence criteria of the solution algorithm are set so as to ensure this error is negligible. Still, the accumulated error cannot be