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Stochastic analysis using the generalized perturbation stable node-based smoothed finite element method

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ABSTRACT

The traditional stochastic finite element method based on the finite element method fails to give the fine solution in precise determination of reliable problems when the computer power consumption is limited. To cure this fatal defect, the generalized *n*th order stochastic perturbation technique based on a stable node-based smoothed finite element method (GS_SNS-FEM) is presented. The framework intends to essentially improve the accuracy, lower the mesh limitation and occupy much less computational consumption for stochastic problems, especially when its second order realization is ineffective for large variations of input random fields. Besides, the *n*th orders expansion makes it possible to get the prefect accuracy for expected values and variances. Numerical examples including the static and dynamic problems are completed and compared with the solution of Monte Carlo simulation. It is found that the SNS-FEM applied in the stochastic problem can improve the accuracy of static and dynamic results, largely decrease the time cost, and lower the requirement of mesh.

1. Introduction

For several decades, the stochastic computational techniques have been implemented using various theoretical and computational methods based on the randomness characters of parameters. The randomness creates the uncertainty in the outputs of the numerical modeling, such as the structure response, temperature and so on. The state-of-the-art review of past, present and future is put forward by George Stefanou in [1] introducing the process of development of stochastic FEM. In early 1988, since FEM is disabled in uncertainty analysis, the concept of stochastic FEM combining with the reliability evaluation based on the FEM has been published in [2]. During last several decades, it proves that stochastic FEM has become a powerful tool in computational stochastic mechanics, especially in large-scale practical engineering problems. It is then introduced into many engineering systems like the aeronautical, mechanical [3,4], civil engineering [5], and even the cracked structure [6,7].

Monte Carlo simulation (MCs) is the simplest and versatile probabilistic method in the framework of stochastic methods. The accuracy of MCs is strongly affected by the number of samples generated by the random number generator. For the purpose of reducing the computational cost, some modified works have been accomplished in [8–10]. However, MCs is still the most stable and robust simulation through sacrificing tremendous time and usually used as the reference solution contrasted with other probabilistic methods [11,12]. Gradually, with the development of stochastic finite element, the spectral stochastic finite element method (SSFEM) has been presented by Ghanem and Spanos [13] as the extension of the deterministic finite element method. And the spread of the spectral stochastic method has also promoted the development of spectral stochastic finite element method [14–17]. The Taylor series expansion of stochastic finite element known as the perturbation method is recommended in the literature [18–21]. The generalized 2nd and *n*th order stochastic perturbation technique has been presented by Kamiński in [22] and [23]. An advisable selection of these solution methods can be made by considering the complexity of basic deterministic numerical techniques, time, computer power consumption, and especially the accuracy. The precise determination of reliable problems is still an open problem in computational mechanics. In order to acquire the satisfactory accuracy, the *n*th order stochastic perturbation technique has been applied into precise problems by sacrificing the time in Kamiński's work. However, the research is just based on the traditional finite element method (FEM). Though the higher order stochastic perturbation technique is employed to get the fine result, the potential defect of low accuracy in traditional FEM is occurred to greatly influence and even eliminate the high accuracy which brought by the *n*th order technique. As a consequence, it

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leads to the inaccuracy of reliable analysis of precise determination even the higher expansion technique is used. It is the wise way to largely increase the DOFs of model to solve this error stemming from basic FEM numerical technique. However, it inevitably leads to much more computational cost. It is necessary and vital to introduce special numerical method balancing the relationship between accuracy and computational cost. In some practical occasions, especially in complex structure, the result of FEM is seriously deteriorated caused by the easily-distorted mesh character. In other words, it is the limitation that the result heavily relies on the quality of mesh.

Many more modified numerical methods have appeared to amend the shortcomings of FEM. Liu et al. [24] applied the cell-based smoothed finite element method into stochastic problems combining with generalized *n*th order stochastic perturbation. This method can improve the solution accuracy significantly for stochastic problem. However, the basic method of CS-FEM is more suitable to the regular mesh and the dynamic problem is not considered. Besides, the accuracy also has potentials to improve further.

The stable node-based smoothed finite element method(SNS-FEM) is presented to overcome these fatal shortcomings. It elevates the accuracy effectively and resists the mesh distortion particularly in the irregular mesh. Consequently, it is necessary to introduce into the stochastic problem. A strain smoothing stabilization for nodal integration is proposed by Chen [25,26] through providing a stabilized conforming nodal integration to eliminate spatial instability in nodal integration. Gradually, it has been further developed and emerged a generalized smoothed Galerkin (GS-Galerkin) weak form by Liu [27], which is the basic foundation of the node-based smoothed finite element method (NS-FEM) [28]. The node-based smoothed finite element is just a category of the smoothed finite element (SFEM). Based on different types of smoothing domain, it is divided into the edge-based S-FEM, face-based S-FEM and so on. More theoretical expansions of G-space related to the smoothed finite element have been presented by Liu [29,30]. The interesting properties of NS-FEM are emphasized here: (1) When a pretty fine mesh is adopted, NS-FEM will give the upper bound value of exact solution which is known as the character of upper bound. (2) NS-FEM eliminates the phenomenon of volumetric locking. (3) It can adjust to polygonal elements with an arbitrary number sides. (4) The domain can be discretized in more flexible way, in other words, the distorted element can not affect the results severely. However, NS-FEM still exists the fatal issue—temporal instability which is defined that models have the spurious non-zero eigen modes. For transient dynamic problem, unphysical numerical responses can't be avoided even the unconditionally stable time-integration way is used. Beissel and Belytschko [31] proposed a stabilized nodal integration procedure adding the residue item to the potential energy function. Then Zhang et al. [32], Feng et al. [33] and Wang et al. [34] further translated this way to the NS-FEM solutions. But it still has its stubborn defects that an effective stabilization parameter of a properαvalue is hard to select. Puso [35] has presented an effective nodal integration technique by increasing the integration points to solve the instability and provide a significant reference for the further studies.

The stable nodal integration method with strain gradient has been presented by Feng [36] and Wang [37–39]. Firstly, the operation of strain smoothing is carried out within every smoothing domain in the same manner of NS-FEM. Then the strains are expanded as same as the method of Taylor equation. Consequently, the sum of energy encompassing several-domains is corrected. In this way, the stable node-based smoothed finite element method (SNS-FEM) is also formulated. SNS-FEM inherits the pretty property and eliminates the temporal instability especially in transient dynamic problem. For elastic-static problems, SNS-FEM can give better and more accurate results in displacement field than other reference solutions and provide accurate stress results. In addition, concerning free vibration analysis, the spurious non-zero energy modes are eradicated successfully and the accurate natural frequency values can be acquired.

In this paper, SNS-FEM is introduced into stochastic problems because of its own prefect properties. A generalized stochastic stable node-based smoothed finite element method (GS_SNS-FEM) is presented for elasto-statics and free vibration problems of solid mechanic. In GS_SNS-FEM, even the coarse mesh using three-node triangular can achieve the accurate deterministic result and save a lot time in stochastic analysis. Then accuracy and time cost in elasto-static analysis and free vibration analysis are studied in detail. Three numerical examples are presented to demonstrate advantages of present method, in comparison with the Monte Carlo simulation.

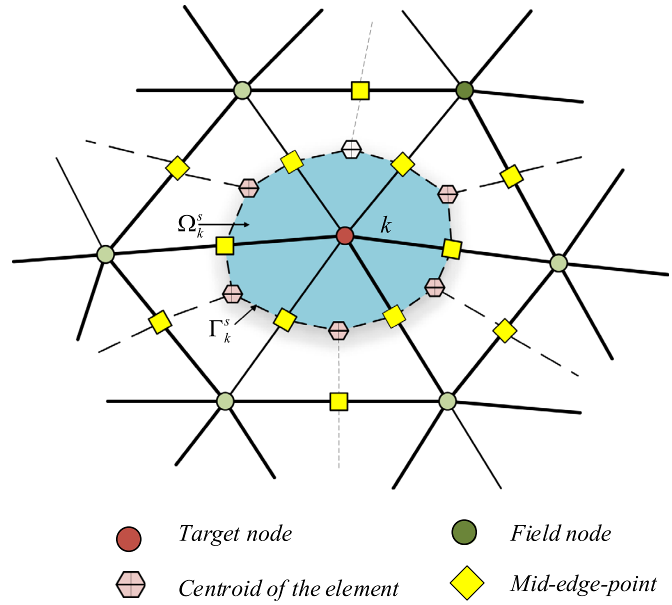

Fig. 1. The schematic of a node-based smoothing domain for node *k*.

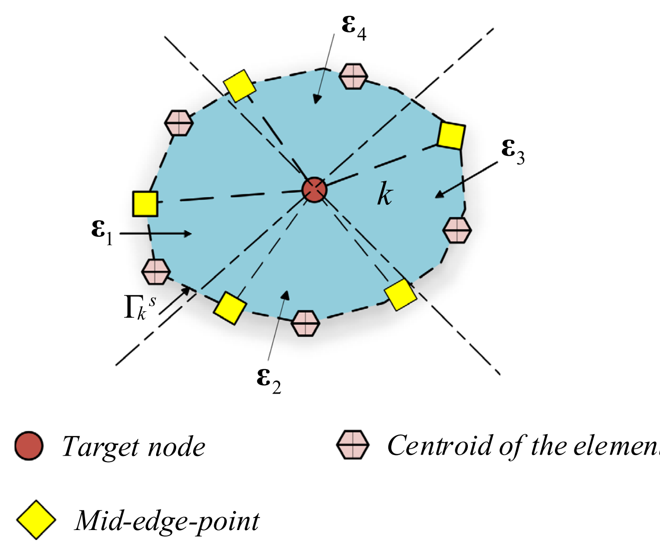

Fig. 2. The smoothing domain is further divided into four sub-domains.