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Sufficient optimality conditions for distributed, non-predictive ramp metering in the monotonic cell transmission model

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ABSTRACT

We consider the ramp metering problem for a freeway stretch modeled by the Cell Transmission Model. Assuming perfect model knowledge and perfect traffic demand prediction, the ramp metering problem can be cast as a finite horizon optimal control problem with the objective of minimizing the Total Time Spent, i.e., the sum of the travel times of all drivers. For this reason, the application of Model Predictive Control (MPC) to the ramp metering problem has been proposed. However, practical tests on freeways show that MPC may not outperform simple, distributed feedback policies. Until now, a theoretical justification for this empirical observation was lacking. This work compares the performance of distributed, non-predictive policies to the optimal solution in an idealised setting, specifically, for monotonic traffic dynamics and assuming perfect model knowledge. To do so, we suggest a distributed, non-predictive policy and derive sufficient optimality conditions for the minimization of the Total Time Spent via monotonicity arguments. In a case study based on real-world traffic data, we demonstrate that these optimality conditions are only rarely violated. Moreover, we observe that the suboptimality resulting from such infrequent violations appears to be negligible. We complement this analysis with simulations in non-ideal settings, in particular allowing for model mismatch, and argue that Alinea, a successful, distributed ramp metering policy, comes close to the ideal controller both in terms of control behavior and in performance.

1. Introduction

Ramp metering refers to the active control of the inflow of cars on a freeway via the onramps, by means of installing and controlling a traffic light at every onramp. In this work, we consider the freeway ramp metering problem over a finite horizon, e.g. one day or one rush-hour period. Freeway ramp metering has been established as an effective and practically useful tool to improve traffic flows on congestion-prone freeways (Papageorgiou et al., 2003; Papageorgiou and Kotsialos, 2000). We study the problem by adopting the Cell Transmission Model (CTM) for freeways, as introduced in the seminal work by Daganzo (1994; 1995), which can be interpreted as a first-order Godunov approximation of the continuous Lighthill–Whitham–Richards-model (LWR) (Lighthill and Whitham, 1955; Richards, 1956). Modifications and generalizations of this model have since been introduced, and we consider a slight generalization of the original, piece-wise affine CTM to a more general, concave fundamental diagram (Como et al., 2016; Coogan and Arcak, 2016; Lovisari et al., 2014). Furthermore, we employ a simplified onramp model originally introduced as the "asymmetric" CTM

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(Gomes and Horowitz, 2006; Gomes et al., 2008), which simplifies the model of onramp-mainline merges by distinguishing mainline- and onramp-traffic demand.

The popularity of the CTM for model-based control stems from the simplicity of the model equations, allowing for computationally efficient solution methods for finite horizon optimal control problems. In particular, relaxing the piecewise-affine fundamental diagram allows one to pose such optimal control problems as linear programs (Ziliaskopoulos, 2000). The solution of the relaxed problem can be made feasible for the original, non-relaxed problem by employing mainline demand control (Muralidharan and Horowitz, 2012). In addition, conditions on the structure of the road network and its dynamics have been derived, which ensure that ramp metering alone is sufficient to make solutions feasible (Gomes and Horowitz, 2006; Como et al., 2016).

A conceptually different approach uses distributed feedback. In such ramp metering policies, local controllers only receive measurements from sensors in close vicinity to any particular onramp and only exchange limited amounts of information, if at all (Papageorgiou et al., 1991; Stephanedes, 1994; Zhang and Ritchie, 1997). Those control policies aim at maximizing bottleneck flows locally, but have been shown to come close to the performance of optimal ramp metering policies in real-world evaluations (Smaragdis et al., 2004; Wang et al., 2014; Papamichail et al., 2010b). While it is apparent that such local feedback controllers are far easier to implement than model-based optimal control policies, it is not obvious why and when the performance of distributed, non-predictive ramp metering policies comes close to the centralized, optimal control solution. A special case for which the optimal control policy can be explicitly constructed is analyzed in (Zhang and Levinson, 2004). It is stated that the structure of the explicit solution "explains why some local metering algorithms [...] are successful – they are really close to the most-efficient logic". However, no proof of optimality is provided. An interesting result exists for the problem of controlling a network of signalized intersections, modeled as a queue network. In particular, throughput optimality of a distributed controller, the so-called max-pressure controller, has been shown via network calculus arguments (Varaiya, 2013). However, the employed network model does not include congestion spill-back effects.

In addition, distributed policies are popular in routing problems, where the route choice through a network is (partially) controlled. It was established that monotonicity of certain routing policies can be leveraged in the analysis of its robustness with respect to non-anticipated reductions of the capacity of individual links (Como et al., 2013b; 2013a). Subsequently, a generalized class of monotonic, distributed routing policies which implicitly back-propagate congestion effects was introduced and it was proven that such policies stabilize an equilibrium that maximizes throughput (Como et al., 2015), for dynamical network models reminiscent of the CTM. Even though we will not assume that split ratios can be actuated in this work, it is interesting to note that monotonicity of parts of the system dynamics will also be essential to our analysis.

This work addresses the question of how distributed, non-predictive ramp metering policies compare to optimization-based, centralized and predictive control policies, in terms of minimization of the Total Time Spent (TTS) for a freeway stretch modeled by the monotonic CTM. If perfect model knowledge and perfect traffic demand prediction is assumed, the ramp metering problem can be cast as a finite horizon optimal control problem, which can be reformulated as a Linear Program in case of a piecewise-affine fundamental diagram (Gomes and Horowitz, 2006; Como et al., 2016). However, both perfect traffic demand prediction and perfect model knowledge are unrealistic (Burger et al., 2013). We retain the assumption of perfect model knowledge for the purpose of the theoretical analysis but drop the assumption of prediction of external demands and introduce a distributed, non-anticipative feedback controller. This controller can be motivated as a one-step-ahead maximization of local traffic flows and is hence called the best-effort controller. We proceed to derive conditions under which such a controller performs optimally, that is, minimizes TTS. To demonstrate the applicability of our results, we perform a simulation case study based on a real-world freeway described in (de Wit et al., 2015), using the monotonic CTM with freeway parameters and demand profiles estimated from real measurements. The simulations confirm the theoretical results as can be seen on days when the optimality conditions are satisfied at all time steps. However, our results do not provide any a priori bound on the suboptimality of the solution even for small violations of the conditions. Nevertheless, even on days during which violations of the optimality conditions are observed, a-posteriori suboptimality bounds can be computed by employing results from the theoretical analysis. In the evaluation, we find that violations of the optimality conditions affect only a small number of time steps for any given day. We also observe that such infrequent violations tend to lead to negligible optimality gaps, although this can only be verified a-posteriori.

The main contribution of this work lies in the derivation of sufficient optimality conditions for minimal TTS ramp metering in the monotonic CTM, which provide a theoretical explanation of why and when the performance of distributed, non-predictive ramp metering and optimal control policies coincides. The assumption of perfect model knowledge employed for the sake of the theoretical analysis is not practical, of course, but we proceed to show that the best-effort controller can in turn be interpreted as an idealized version of the successful Alinea controller (Papageorgiou et al., 1991), which replaces the need for perfect model knowledge by virtue of feedback.

Note that the analysis in this work is applicable to the monotonic CTM as studied in e.g. (Gomes and Horowitz, 2006; Como et al., 2016). Empirical evidence suggests the existence of a non-monotonic capacity drop of freeway traffic flow in congestion and (first-order) models incorporating this effect have been proposed (Kontorinaki et al., 2016; Yuan et al., 2017). However, so far no general, efficient solution method (e.g. based on a convex reformulation) of optimal control problems for the non-monotonic CTM is known and this work does not directly generalize to a non-monotonic setting. Rather, it should be viewed as an intermediate step towards solving the more general problem.

The paper is organized as follows: Section 2 defines the CTM and the main problem of minimizing TTS. In Section 3, we introduce the non-predictive, distributed best-effort controller. We also show that this controller is not always optimal, by