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Representation requirements for perfect first-in-first-out verification in continuous flow dynamic models

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ABSTRACT

Dynamic models of traffic require answers for many issues. One of them is the way priorities of different traffic streams (commodities) are managed. This is particularly challenging when flows are treated as continuous. It is common to consider the First-In-First-Out (FIFO) rule as a baseline for setting priorities; but most practical continuous flow dynamic models do not satisfy FIFO perfectly. This paper examines the difficulties associated with full adherence to network-wide FIFO.

We examine six different ways to represent dynamic flow solutions over a network, and for each representation we discuss whether it is sufficient for verifying FIFO, whether the verification process is finite, and whether proving FIFO can be directly implied. Throughout the evaluation eight alternative definitions of FIFO are considered, seven of them are shown to be essentially equivalent, while the last definition is not, and may therefore be considered as “weak” FIFO. The most promising representation appears to be the one denoted as “cohort bundles,” while somewhat more abstract than the other representations, supporting this representation directly shows perfect FIFO satisfaction. Further evaluation of this representation remains a subject for future research.

In a nutshell, the key conclusion of this analysis is that in order to satisfy perfect network-wide FIFO the number of discretized elements of flow should probably be allowed to grow quickly and unboundedly with model duration, and it cannot be determined a-priori. These insights about the challenges of incorporating FIFO in continuous flow dynamic models, which may be relevant also for other behavior-based priority rules, can help modelers and practitioners set realistic expectations regarding the level of control over priority rules that can be achieved within finite-dimensional continuous flow dynamic models.

1. Introduction

Continuous flow dynamic network loading (DNL) models have been studied extensively in recent decades, and proved useful in many practical applications. Popular examples are the cell transmission model – CTM (Daganzo, 1994) and the link transmission model – LTM (Yperman et al., 2006). In these models the propagation of traffic through nodes (from cell to cell or from link to link) is depicted by continuous variables representing the amount of traffic crossing the node

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within discrete time steps. DNL variants differ in several main aspects: grid discretization vs. grid-free methods; link model; node model or intersection representation (merges, diverges, as well as signalized and unsignalized intersections); aggregation/disaggregation of flow into commodities, e.g. by origin/destination/route; and priority rules governing the management (or book-keeping) of commodities by priorities at the link and node levels. This study focuses on the last aspect, namely how priorities of commodities are managed.

In many previous studies, as early as Carey (1986) and Smith (1993), flow propagation priority rules aimed to satisfy the first-in-first-out (FIFO) condition. When time and space are considered as continuous, conditions of perfect network-wide FIFO can be formulated (e.g. Friesz et al., 1993; Friesz et al., 2013). Practical numerical solutions must have a finite-dimensional representation, i.e. some form of discretization.[1] One of the most elaborate schemes for handling network-wide FIFO in finite-dimensional solutions is by consideration of history lists (Smith, 1993). The scope of this paper is exploring the range of solution representations between the two options: fully continuous and history lists. Formal expositions of these two solution representations are given in Sections 3 and 8 respectively, and for the other representations in Sections 4–7.

History lists may seem to be as far as one would wish to go in an effort to satisfy FIFO. However, as we shall see, history lists are not sufficient for perfect FIFO verification. To the best of our knowledge, with a few exceptions (e.g. Smith, 1993), most finite-dimensional continuous-flow DNL models with route departure rate inputs address FIFO only within links, at various levels of precision or approximation. Therefore, these models are further away from perfect network-wide FIFO than solutions with history list representations. Clearly many of these models are very useful and have important value in dealing with the complexity of dynamic traffic networks, despite their deviation from FIFO. Future research stemming from the present paper may enable producing perfect FIFO solutions, at least for specific cases. Such solutions could be used to evaluate the magnitude of the deviations from FIFO in alternative approximate models, and to explore the practical implications of these deviations on model usefulness.

A few previous studies examined part of the difficulties associated with implementing FIFO. One study illustrated how commonly used methods in CTM may lead to FIFO deviations (Blumberg and Bar Gera, 2009). A subsequent study (Carey et al., 2014) explored several levels of FIFO approximation, demonstrated why perfect FIFO cannot be ensured by any of them, thus further illustrating the difficulty of dealing with the FIFO condition.

It is well known that in real traffic adherence to FIFO is not perfect. Some may argue that if our models deviate from FIFO and reality deviates from FIFO then there is no problem. However, there is no a-priori reason to assume that the deviations from FIFO in models (stemming from computational reasons) should bear resemblance to the deviations from FIFO in real traffic (stemming from driver behavior). In addition, there is no reason to assume that alternative priority rules will be easier than FIFO to implement computationally, especially if such rules will be derived from quantitative empirical evidence to reflect actual traffic behavior.

It seems reasonable to expect that modelers should have control over the priority rule in their model. In particular, modelers may expect the freedom to choose whether FIFO will be satisfied strictly, or whether to deviate from FIFO. In the latter case modelers may wish to specify the nature and the magnitude of the deviations from FIFO they wish to introduce. This is why we believe that better understanding of the FIFO challenges is important, a goal we hope to partially address in this paper.

To achieve this goal, two approaches can be considered: (1) a constructive practical approach focusing on what needs to be done during the computation of, e.g., a CTM solution; and (2) a formalistic approach focusing on definitions of mathematical conditions that can be used to verify perfect FIFO, and particularly what are the requirements from the way the solution is described in order to ensure perfect FIFO. A constructive approach will be developed in a separate article. The focus of this article is the second approach.

By FIFO verification, in this paper, we are concerned with what numerical tests or checks would need to be undertaken (explicitly or implicitly) to determine whether a given DNL solution does, or does not, fully satisfy FIFO. In some cases this could mean checking the solution at every point in time and space. In other cases, it may be sufficient to check the solution at a finite number of points, e.g. if we can assume linearity of time-space trajectories between these discrete points. Note that the DNL solutions are taken as given, already constructed, hence we are not concerned here with methods used to construct DNL solutions. Also, this paper is not concerned with taking individual DNL models or methods and proving whether they do, or do not, always fully satisfy FIFO.

The perspective taken in this paper is not a standard one. It can be viewed as a thought experiment involving an interface between two participants: one participant is the solution proponent, and the other is the FIFO verifier. The FIFO verifier asks the solution proponent to compute various values, and based on these values she tries to identify FIFO violations, or to determine that the solution satisfies network-wide FIFO perfectly. For example, the FIFO verifier may ask: “What is the cumulative volume at time $t = 37\,\mathrm{s}$ in the position $x = 591\,\mathrm{m}$ along link 22”. The proponent will need to provide a value, say 932.534. The verifier is allowed to submit many queries, and to choose new queries based on the responses to previous queries. The proponent cannot take back any previous answers, and must respond to each query independently of any prior

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[1] The term discretization is used here as in Finite Element Analysis (e.g. Johnson, 1987) to imply any strategy for identifying a mesh, i.e. a finite set of points in the time-space domain, which is used as a basis for choosing a finite-dimensional subset from the infinite-dimensional solution space of continuous functions. The mesh points need not be regular, and they need not be predetermined. Discretization may lead to approximation, but in some cases it can support exact solutions as well.