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A branch-and-price approach for solving the train unit scheduling problem

Zhiyuan Lin*, Raymond S.K. Kwan

School of Computing, University of Leeds, Leeds, LS2 9JT, United Kingdom

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ABSTRACT

We propose a branch-and-price approach for solving the integer multicommodity flow model for the network-level train unit scheduling problem (TUSP). Given a train operator's fixed timetable and a fleet of train units of different types, the TUSP aims at determining an assignment plan such that each train trip in the timetable is appropriately covered by a single or coupled train units. The TUSP is challenging due to its complex nature. Our branch-and-price approach includes a branching system with multiple branching rules for satisfying real-world requirements that are difficult to realize by linear constraints, such as unit type coupling compatibility relations and locations banned for coupling/decoupling. The approach also benefits from an adaptive node selection method, a column inheritance strategy and a feature of estimated upper bounds with node reservation functions. The branch-and-price solver designed for TUSP is capable of handling instances of up to about 500 train trips. Computational experiments were conducted based on real-world problem instances from First ScotRail. The results are satisfied by rail practitioners and are generally competitive or better than the manual ones.

1. Introduction

1.1. Train unit scheduling problem

A *train unit* is a self-propelled non-splittable fixed set of train *cars* (carriages), which is the most commonly used passenger rolling stock in the UK and many other European countries. A train unit is able to move in both directions on its own. Train units are classified into types having different characteristics. A train unit can be coupled with other units of the same or compatible types.

Given a rail operator's timetable on one operational day, a fleet of train units of different types and a rail network of routes, stations and infrastructures, the *train unit scheduling problem* (TUSP) (Lin and Kwan, 2014) refers to the planning of how timetabled trains are covered by a single or coupled train units from the fleet. From the perspective of a train unit, scheduling assigns a sequence of trains to it as its daily workload. A route refers to a unique path in a rail network between two locations. The TUSP may also include auxiliary activities, e.g. empty-running insertion, coupling/decoupling control, platform assignment, platform/siding/depot capacity control, re-platforming, reverse, shunting movements from/to sidings or depots and unit blockage resolution. Capacity control is needed when there are several train units staying at the same

* Corresponding author.
E-mail addresses: Z.Lin@leeds.ac.uk (Z. Lin), R.S.Kwan@leeds.ac.uk (R.S.K. Kwan).

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berthing place such as a platform/siding/depot and it is to make sure the capacity of the berthing place is not exceeded. Re-platforming refers to the operation to shunt a train unit from its arrival platform to a different departure platform. Reverse activities usually occur at a dead-end platform such that the train unit will depart in the opposite direction to its arrival. A reverse activity will change the unit permutation if a train is formed by coupled units, where the unit permutation refers to the order of units (e.g. front, middle, rear) in a coupled formation. More details on the train unit operation rules can be found in Lin and Kwan (2014). In some cases, maintenance and unit overnight balance planning (or unit cycles) are also included; however in the UK, they are often achieved at later stages after train unit scheduling. A similar problem in the literature to the TUSP is the *train unit assignment problem* (TUAP) (Cacchiani et al., 2010b; 2012a; 2013b). Another relevant problem is *rolling stock circulation problem* (Alfieri et al., 2006; Peeters and Kroon, 2008; Schrijver, 1993), which however has different problem definitions than TUAP and TUSP.

1.2. A branch-and-price solver for TUSP

This paper addresses the particular issue of designing an efficient branch-and-price ILP solver for the integer multicommodity flow (IMCF) models used for the network-level TUSP. An IMCF model is proposed in Cacchiani et al. (2010b) for the TUAP, where the integer program is solved by an LP-based diving heuristic. A two-phase approach is presented in Lin and Kwan (2014) for the TUSP. The first phase is a fixed-charge IMCF model, where for most tested real-world instances the integer solver only generated columns at the root node in the BB tree, such that the solution qualities are not guaranteed to be exact. Although exact methods are used for the rolling stock circulation problems, the problem definitions are different from Cacchiani et al. (2010b) or Lin and Kwan (2014). For the TUAP/TUSP, there is no exact solution method in literature that is able to handle realistic problem sizes especially in the UK as far as the authors are aware.

A branch-and-price solver proposed in Lin and Kwan (2014) mainly uses columns from root node in the BB tree in most tested instances because of its limited capability. No customized branching or node selection strategy is used in the solver. No method is used to strengthen the weak LP-relaxation. The node-family variables are also not capable in handling all possible combination-specific coupling upper bound requirements. Compared with the aforementioned solver, significant improvements have been made in the proposed branch-and-price approach. It is capable of handling real-world instances of up to around 500 trains. The improvements include: (i) A branching system with multiple branching rules including two customized rules as train-family branching and banned location branching; (ii) An adaptive node selection method; (iii) A column inheritance strategy; (iv) A branch-and-bound (BB) system with estimated upper bounds and tree node reservation, and (v) the use of convex hulls for satisfying difficult real-life constraints.

Computational experiments were carried out based on real-world instances from First ScotRail. The solutions often outperform the manual schedules in many aspects and the practitioners from ScotRail are satisfied. Post-processing using TRACS-RS (Tracsis plc, 2016) to resolve the station-level issues like unit blockage and redundant coupling/decoupling is also shown to be successful for the instances tested.

1.3. Contributions

The main contribution of this paper is the design of a customized branch-and-price solver for solving the TUSP. While the TUAP studied by Cacchiani et al.shares similar features of TUSP, the solution approaches for the TUAP are all based on heuristics. For instance, in Cacchiani et al. (2010b), an LP based diving heuristics is used for solving the integer multicommodity flow ILP model. In Cacchiani et al. (2013b), a fast heuristics is applied in conjunction with Lagrangian relaxation. Moreover, compared with the TUAP, additional real-life requirements that are crucial in UK railway operations are considered in the TUSP, such as train unit type compatibility relations, locations banned for coupling/decoupling and combination-specific coupling upper bounds. Customized strategies to accommodate those requirements have been implemented in the branch-and-price solver, such as local convex hulls, branching on train families and banned locations. Finally, some advanced techniques are used in the solver for speeding up the solution process, such as an adaptive node selection method combined with best-first and depth-first, column inheritance and estimated upper bound. Computational experiments have proved their usefulness for the tested real-world instances.

It is worth mentioning that some constraints considered in Cacchiani et al. (2010b), such as maintenance and overnight balance, are not included in our solver. This is because they are conventionally regarded as separate stages in the UK railway industry. Moreover, in Cacchiani et al. (2010b, 2013b), larger numbers of different unit types (up to 10 for real-world instances and 20 for realistic instances) are experimented. In our work, up to three different types are experimented, which is based on the real-world operational rules of the operator's network.

1.4. Organization of the paper

The remainder of this paper is organized as follows. Section 2 surveys the relevant literature. Section 3 describes the TUSP in more details. Section 4 presents the model and integer linear programming formulation. Section 5 gives the branch-and-price approach for solving the above formulation. Section 6 reports computational experiments based on real-world instances. Finally we conclude this paper in Section 7.