Engineering Analysis with Boundary Elements 42 (2014) 45–50

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

A two-dimensional base force element method using concave polygonal mesh

Yijiang Peng*, Lijuan Zhang, Jiwei Pu, Qing Guo

Key Laboratory of Urban Security and Disaster Engineering, Ministry of Education, Beijing University of Technology, Beijing 100124, China

ARTICLE INFO

Article history:
Received 7 April 2013
Accepted 6 September 2013
Available online 15 October 2013

Keywords:
Complementary energy
Finite element method
Base force element method
Concave polygonal meshes

ABSTRACT

Using the concept of base forces given by Gao [12] as fundamental variables to describe the stress state of an elastic system, this paper presents an explicit expression of two-dimensional element compliance matrix on the complementary energy principle with concave polygonal meshes. The detailed 2-D formulations of governing equations for the new finite element method—the base force element method (BFEM) are written in terms of the base forces concept using the Lagrange multiplier method. The explicit displacements expression of nodes is given. To verify the model proposed in this paper, a program on the 2-D BFEM using MATLAB language is made and a number of examples on concave polygonal meshes and aberrant meshes are provided to illustrate the BFEM. Good agreement has been obtained comparing numerical results using the proposed BFEM to available theoretical results. The concave polygonal element model can be used efficiently for the elasticity analysis with increasing element aspect ratios and distortion meshes compared with the traditional finite element method (FEM).

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1. Introduction

The finite element method (FEM) is one of the most important numerical methods developed from the 1950s, and it has been the most popular and widely used numerical analysis tool for problems in engineering and science [23,7,1,2,22,20]. However, because of the rapid development of engineering and science, the problems of computational mechanics grow ever more challenging. Facing these new challenges, some shortcomings and limitations of the traditional FEM that are inherent to numerical methods formulated based on meshes or elements are revealed. Especially analyses of some problems, for example, the large deformation, the crack growth with arbitrary and complex paths, the moving boundary problems, the breakage of material with large number of fragments, etc. The root of these problems for traditional FEM is the dependence on elements or meshes with severity.

The attempts to extend and generalize the complementary-energy methods that have been proposed over the last almost 50 yr for small deformation solid mechanics problems have led to the development of several complementary energy principles and corresponding finite element models for non-linear elastic solid/structural mechanics problems [17–19].

Recently, based on the complementary energy principle, a hybrid stress-function element method is proposed by Fu et al. [8] and [3–6] using the mesh-distortion immune technique for developing 8-node plane elements. It starts from the principle of minimum complementary energy, and employs the fundamental analytical solutions of the airy stress function as the trial functions (analytical trial function method). So long as the element edges keep straight, the resulting 8-node element HSF-Q8 can also produce the exact solutions for quadric displacement fields, even the element shape degenerates into triangle and concave quadrangle.

Although the improvement in the finite element method has been made a lot of researches, but the new finite element method for concave polygonal meshes is still a challenge subject and new methods and ideas are needed.

The new finite element method—BFEM is described by the new concept of "base forces". This concept for describing stress state at a point was given by Gao [12], that is called "base forces". It is much simpler than traditional stress tensors in solving some typical large strain problems, for examples, large strain contact problems and the nonlinear Boussinesq problem. By means of the "base forces", the equilibrium equation, boundary condition and elastic law are written in very simple forms. Therefore, many typical large strain problems of elasticity were solved recent years [9–11]. The "base forces" also show advantages in linear elastic problem, by means of which the explicit expressions of stiffness matrix and compliance matrix of the finite element method were given [12]. The applications of the stiffness matrix to the plane problems of elasticity using the four-side plane element and the polygonal element were researched by Peng et al. [15] using the

* Corresponding author. Tel.: +86 13681107181.
E-mail address: pengyijiang@bjut.edu.cn (Y. Peng).

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http://dx.doi.org/10.1016/j.enganabound.2013.09.002

base force element method (BFEM) on the potential energy principle. Using the concept of base forces as state variables, a three-dimensional formulation of the base force element method (BFEM) on complementary energy principle was proposed by Peng and Liu [14] for geometrically nonlinear problems and the new finite element method based on the concept of base forces was called as the base force element method (BFEM) by Peng and Liu [14]. A three-dimensional model of base force element method (BFEM) on complementary energy principle was proposed by Liu and Peng [13] for elasticity problems. The application of 2-D base force element method (BFEM) to geometrically nonlinear analysis was proposed by Peng et al. [16].

The objective of this paper is to present a new two-dimensional formulation of the base forces element method (BFEM) for concave polygonal mesh problems. A two-dimensional model of the BFEM for concave polygonal mesh problems will be derived based on the complementary energy principle. In the present formulations of the BFEM, the "base forces" are treated as unknown variables, and the basic equations are constructed by means of the complementary energy principle. The element equilibrium conditions are fulfilled using the Lagrange multiplier method. Explicit expressions for the control equations are provided, and a procedure of the present method is developed. A number of arbitrary mesh problems are solved using the present formulation, and the results are compared with corresponding analytical solutions.

2. Concept of base force

Consider a two-dimensional domain of solid medium, let $\mathbf{P}$ and $\mathbf{Q}$ denote the position vectors of a point before and after deformation respectively. $\mathbf{x}^\alpha(\alpha=1,2)$ denotes the Lagrangian coordinates. The following two sets of triads are introduced.

$$\mathbf{P}_\alpha=\frac{\partial \mathbf{P}}{\partial x^\alpha},\quad \mathbf{Q}_\alpha=\frac{\partial \mathbf{Q}}{\partial x^\alpha} \tag{1}$$

In order to describe the stress state at a point $\mathbf{Q}$, an element of plane problem is shown in Fig. 1. Define,

$$\mathbf{t}^\alpha=\frac{d\mathbf{t}^\alpha}{dx^{\alpha+1}},\quad dx^\alpha\to 0 \tag{2}$$

in which $\mathbf{t}^\alpha$ is called the base forces at point $\mathbf{Q}$ in the two-dimensional coordinate system x^α. In the above index α have values from 1 to 2 and when $\alpha+1=3$, we promise that $3=1$.

When we use $\mathbf{t}^\alpha$ to describe the stress state, simultaneously we should use its conjugate variables to describe the deformation. Let $\mathbf{u}$ denote the displacement of a point, then

$$\mathbf{Q}=\mathbf{P}+\mathbf{u},\quad \mathbf{Q}_\alpha=\mathbf{P}_\alpha+\mathbf{u}_\alpha \tag{3}$$

Where

$$\mathbf{u}_\alpha=\frac{\partial \mathbf{u}}{\partial x^\alpha} \tag{4}$$

$\mathbf{u}_\alpha$ is called the displacement gradients.

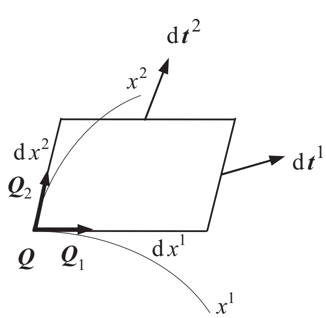

Fig. 1. Base forces.

Further let

$$A_P=|\mathbf{P}_1\times\mathbf{P}_2|,\quad A_Q=|\mathbf{Q}_1\times\mathbf{Q}_2| \tag{5}$$

where A_P and A_Q are called the base areas before and after deformation respectively.

Let W be the strain energy per unit mass and $\mathbf{u}$ gain an increment $\delta\mathbf{u}$, using the virtual work principle, we can directly obtain the following relation:

$$\mathbf{t}^\alpha\delta\mathbf{u}_\alpha=\rho A_Q\delta W=\rho_0 A_P\delta W \tag{6}$$

in which ρ_0 and ρ are called the mass density before and after deformation respectively.

Then

$$\mathbf{t}^\alpha=\rho A_Q\frac{\partial W}{\partial \mathbf{u}_\alpha}=\rho_0 A_P\frac{\partial W}{\partial \mathbf{u}_\alpha} \tag{7}$$

Eq. (7) directly expresses the $\mathbf{t}^\alpha$ by strain energy. Thus, $\mathbf{u}_\alpha$ is just the conjugate variable of $\mathbf{t}^\alpha$. From Eq. (7), we can see the mechanics problem can be completely established by means of $\mathbf{t}^\alpha$ and $\mathbf{u}_\alpha$.

Accordingly to the definitions of various stress tensors, we can give the relation between the base forces and various stress tensors. The Cauchy stress is

$$\boldsymbol{\sigma}=\frac{1}{A_Q}\mathbf{t}^\alpha\otimes\mathbf{Q}_\alpha \tag{8}$$

where $\otimes$ the dyadic symbol, and the summation rule is implied.

The first Piola–Kirchhoff stress is

$$\boldsymbol{\tau}=\frac{1}{A_P}\mathbf{t}^\alpha\otimes\mathbf{P}_\alpha \tag{9}$$

The second Piola–Kirchhoff stress is

$$\boldsymbol{\Sigma}=\frac{1}{A_P}\mathbf{t}_P^\alpha\otimes\mathbf{P}_\alpha \tag{10}$$

in which

$$\mathbf{t}_P^\alpha=\mathbf{F}^{-1}\mathbf{T}^\alpha \tag{11}$$

where $\mathbf{F}$ is called the triads transfer tensor

$$\mathbf{F}^{-1}=\mathbf{P}_\alpha\otimes\mathbf{Q}^\alpha \tag{12}$$

where $\mathbf{Q}^\alpha$ is the conjugate vector of $\mathbf{Q}_\alpha$.

For the isotropic material, the complementary energy density can be expressed as follow:

$$W_C=\frac{1}{2E}[(1+\nu)J_{2T}-\nu J_{1T}^2] \tag{13}$$

in which E is Young's modulus, ν is Poisson's ratio, J_{1T} and J_{2T} are the invariants of $\mathbf{t}^\alpha$, so

$$J_{1T}=\frac{1}{A_P}\mathbf{t}^\alpha\mathbf{P}_\alpha\quad ,\quad J_{2T}=\frac{1}{A_P^2}(\mathbf{t}^\alpha\mathbf{t}^\beta)p_{\alpha\beta} \tag{14}$$

Where $p_{\alpha\beta}$ is a metric tensor in the original configurations, and there are the following relations:

$$p_{\alpha\beta}=\mathbf{P}_\alpha\mathbf{P}_\beta \tag{15}$$

3. Development of 2-D BFEM with concave polygonal meshes

Now, consider a concave polygonal element as shown in Fig. 2. Let I, J, K, L, M, N denote its edges, and $\mathbf{T}^I, \mathbf{T}^J, \mathbf{T}^K, \mathbf{T}^L, \mathbf{T}^M, \mathbf{T}^N$ the force vectors acting on each of the edges.

Substituting Eq. (14) into Eq. (13), the complementary energy of an element can be reduced as follows:

$$W_C^e=\frac{1+\nu}{2EA}\left[(\mathbf{T}^I\mathbf{T}^J)p_{IJ}-\frac{\nu}{1+\nu}(\mathbf{T}^I\mathbf{P}_\mathbf{I})^2\right] \tag{16}$$

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