Engineering Analysis with Boundary Elements 42 (2014) 51–59

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

A regularized Lagrangian meshfree method for rainfall infiltration triggered slope failure analysis

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ARTICLE INFO

Article history:
Received 6 April 2013
Accepted 3 September 2013
Available online 2 October 2013

Keywords:
Regularized meshfree method
Two-level Lagrangian strain smoothing
Slope failure
Rainfall infiltration
Stabilized conforming nodal integration

ABSTRACT

A two-level Lagrangian strain smoothing regularized meshfree formulation based on the stabilized conforming nodal integration is developed for analyzing the rainfall infiltration triggered large deformation slope failure. The regularization is fulfilled by developing two-level smoothed nodal gradients of the meshfree shape function in the Lagrangian meshfree discretization of the weak form of the coupled soil-rainfall infiltration equations. The two-level smoothed nodal gradients of the meshfree shape function are successively constructed based on the one-level smoothed meshfree nodal gradients which are obtained through a Lagrangian gradient smoothing operation on the deformation gradient and the rate of deformation tensor. The slope failure initiation and evolution is realized by a coupled isotropic damage and pressure-dependent plasticity model. Numerical results demonstrate that the present method is very effective for modeling large deformation slope failure process induced by rainfall infiltration.

1. Introduction

Plenty of practical observations and engineering practice have shown that the rainfall infiltration triggered slope failure is one of the major natural disasters that may lead to severe loss of life and property [1]. To effectively predict and mitigate the rainfall infiltration triggered failure, significant research effort has been spent on the analytical, experimental and computational investigation on the mechanism of this type of slope failure [2]. As a result it has been widely recognized that the interaction between the water and soil skeleton in the unsaturated soil slope plays a central role for the damage evolution in the slope. Due to the coupled and complex mechanical nature of this type of slope failure, it is not an easy task to perform the laboratory tests as well as analytical study. On the other hand, the computer simulation has gained growing attention for investigation of slope failure and the most frequently used numerical method is the finite element method [3]. However due to the topological mesh connectivity the conventional finite element method is not very effective for large deformation problems with severe mesh distortion, evolving boundary problems like crack propagation, discretization sensitive strain localization problems, and the higher order problems with global C^1 continuity requirement like thin plate and shell problems.

Consequently since 1990s, a new generation of computational methods, the so-called meshfree or meshless methods, have received considerable attention with a wide range of applications and many versatile methods have been developed. A review on the state of the art of the meshfree methods and their applications can be found in [4–8] and the references therein. In this work the Galerkin type of meshfree method such as the element free Galerkin method [9] or the reproducing kernel particle method [10] based on the moving least square (MLS) or reproducing kernel (RK) approximation is employed. It turns out that MLS and RK approximations are identical when the monomial basis functions are used. The MLS/RK approximation is constructed purely based on nodal information and is very suitable for large deformation analysis. This advantage is fully illustrated by Chen et al. [11] by effectively solving various path-independent and path-dependent benchmark problems and subsequently Wu et al. [12] introduced a Lagrangian–Galerkin meshfree formulation for large deformation analysis of geotechnical materials.

Despite of the rapid developments and applications, the meshfree method often suffers from the low computational efficiency because of the non-polynominal characteristic of MLS/RK meshfree shape function that necessitates higher order quadrature rule for the numerical weak form integration. To improve the efficiency the residual of equilibrium equation with a scaling parameter was added to the potential functional for stabilization by Beissel and Belytschko [13] to construct a stabilized meshfree method with nodal integration. Through the strain smoothing operation Chen et al. [14–16] developed a stabilized conforming nodal integration

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http://dx.doi.org/10.1016/j.enganabound.2013.09.001

(SCNI) method for Galerkin meshfree formulations. This method provides a reasonable balance between the stability and efficiency. Moreover it meets the linear exactness condition and thus exhibits high solution accuracy. Later this approach was systematically developed to 2D and 3D nonlinear problems [16–18] and the beam, plate and shell problems [19–26]. The stability of SCNI-based meshfree method was also studied in detail by Chen and Wu [27]. Chen et al. [28] presented a comprehensive summary about the stabilized conforming nodal integration methodology. Recently within the stabilized conforming nodal integration framework, Wang et al. [29] proposed an efficient two-level nodal strain smoothing meshfree approach to resolve the discretization sensitivity issue in damage analysis, which successively employs the strain smoothing method developed by Chen et al. [14] for material instability regularization.

In this study a two-level Lagrangian strain smoothing meshfree approach with the stabilized conforming nodal integration is developed for large deformation simulation of soil slope failure under rainfall infiltration condition. The coupled equations of soil motion and rainfall seepage are considered in the context of meshfree method. The spatial discretization of the weak form is carried out by a SCNI-based two-level Lagrangian strain smoothing meshfree method and the explicit time integration is performed for the temporal dimension. The initiation and propagation of failure in the soil slope is modeled by the coupled constitutive equations of isotropic damage and Drucker–Prager pressure-dependent plasticity using the net stress measure. Besides, the classical rate formulation is introduced to accommodate the large deformation effect.

The remainder of this paper is organized as follows. In Section 2, the coupled governing equations of soil and rainfall infiltration are presented first and then the constitutive relationships including damage, plasticity and seepage are discussed. Subsequently the Lagrangian MLS/RK approximation based on the material kernel is briefly summarized in Section 3. In Section 4 a SCNI-based two-level Lagrangian strain smoothing regularized meshfree discretization of field equations is developed in detail. In Section 5 numerical results are shown to verify the proposed method, which are followed by the concluding remarks in Section 6.

2. Governing equations

2.1. Momentum balance and continuity equations

Consider a soil medium that occupies a bounded domain Ω_X with boundary Γ_X at the initial configuration, thus the motion of a material point $\mathbf{X}\in\overline{\Omega}_X=\Omega_X\cup\Gamma_X$ can be described by $\mathbf{x}=\boldsymbol{\varphi}(\mathbf{X},t)$, where $\mathbf{x}\in\overline{\Omega}=\Omega\cup\Gamma$ denotes the spatial location corresponding to the material point $\mathbf{X}$, $\boldsymbol{\varphi}(\mathbf{X},t)$ is the deformation mapping function to be determined, Ω and Γ represents the domain and boundary of the current configuration. The governing equations of momentum balance and fluid continuity referring to the current configuration for soil and seepage can be stated as [2,3]

$$\begin{cases} \nabla^S\cdot\boldsymbol{\sigma}^*+\nabla^S p_a+\mathbf{b}=\rho\mathbf{a} & \text{in } \Omega \\ \frac{\partial(\rho_w \hbar S_r)}{\partial t}+\nabla^S\cdot(\rho_w\mathbf{v}_w)=0 & \text{in } \Omega \end{cases} \tag{1}$$

where ∇^S is the spatial gradient operator, $\boldsymbol{\sigma}^*$ is the net stress tensor given by $\boldsymbol{\sigma}^*=\boldsymbol{\sigma}-p_a\mathbf{1}$ and $\boldsymbol{\sigma}$ is the total stress, $\mathbf{1}$ is the second order identity tensor, p_a is the pore air pressure, $\mathbf{b}$ is body force, ρ and ρ_w are the soil–water mixture and water densities, $\mathbf{a}$ is the acceleration of soil, $\hbar$ is the soil porosity, S_r represents the degree of soil saturation, and $\mathbf{v}_w$ is the seepage velocity. The auxiliary boundary and initial conditions to complete the problem statement of Eq. (1) are

$$\begin{cases} \mathbf{u}=\hat{\mathbf{u}} \quad \text{on} \quad \Gamma^g; \quad \boldsymbol{\sigma}\cdot\mathbf{n}=\mathbf{t} \quad \text{on} \quad \Gamma^t; \\ h=\hat{h} \quad \text{on} \quad \Gamma^h; \quad \mathbf{v}_w\cdot\mathbf{n}=q \quad \text{on} \quad \Gamma^w \\ \mathbf{u}(\mathbf{x},0)=\mathbf{u}_0(\mathbf{x}), \quad \dot{\mathbf{u}}(\mathbf{x},0)=\dot{\mathbf{u}}_0(\mathbf{x}), \quad h(\mathbf{x},0)=h_0(\mathbf{x}) \quad \text{in } \Omega \end{cases} \tag{2}$$

where the first and second rows list the essential and natural boundary conditions, and the third row describes the initial boundary conditions. Γ^g and Γ^t represent the essential and natural boundaries, $\mathbf{n}$ is the outward boundary normal. Γ^h is the prescribed water head boundary and Γ^w denotes the flux boundary condition of the rainfall infiltration. $\mathbf{u}$ denotes the soil displacement and h is the water head. $\hat{\mathbf{u}}$, $\mathbf{t}$, $\hat{h}$, q, $\mathbf{u}_0$, $\dot{\mathbf{u}}_0$, h_0 are the corresponding given boundary and initial values, respectively.

Without the consideration of the air density, the density ρ of the soil–water mixture can be expressed as

$$\rho=(1-\hbar)\rho_s+\hbar S_r\rho_w \tag{3}$$

where ρ_s is the density of soil skeleton, the soil porosity $\hbar$ is related to the void ratio ϵ as $\hbar=\epsilon/(1+\epsilon)$. From Eq. (3), it is noticed that the soil density ρ is a function of the degree of soil saturation S_r and the porosity $\hbar$.

The updated Lagrangian variational statement of the system of equations in (1) can be written as

$$\begin{cases} \int_\Omega \delta\mathbf{u}\cdot\rho\mathbf{a}d\Omega=\int_\Omega\delta\mathbf{u}\cdot\mathbf{b}d\Omega+\int_{\Gamma^h}\delta\mathbf{u}\cdot\mathbf{t}d\Gamma-\int_\Omega(\nabla^{Sym}\delta\mathbf{u}):\boldsymbol{\sigma}^*d\Omega \\ \int_\Omega\delta p_w\hbar S_{r,s}\dot{p}_w d\Omega=\int_{\Gamma^w}\delta p_w q d\Gamma+\int_\Omega\delta p_w \operatorname{tr}(\dot{\boldsymbol{\varepsilon}})S_r d\Omega-\int_\Omega(\nabla^S\delta p_w)\cdot\mathbf{v}_w d\Omega \end{cases} \tag{4}$$

where p_w is the water pore pressure. ∇^{Sym} represents the symmetric spatial gradient operator, $\dot{\boldsymbol{\varepsilon}}$ is the rate of deformation tensor. $S_{r,s}=\partial S_r/\partial s$, and s denotes the suction which is given by $s=p_a-p_w$, tr is the trace operator. The rate of deformation tensor $\dot{\boldsymbol{\varepsilon}}$ and the spin tensor $\boldsymbol{\omega}$ are related to the velocity $\dot{\mathbf{u}}$ as

$$\begin{cases} \dot{\boldsymbol{\varepsilon}}=\frac{1}{2}(\dot{\mathbf{u}}\otimes\nabla+\nabla\otimes\dot{\mathbf{u}}) \\ \boldsymbol{\omega}=\frac{1}{2}(\dot{\mathbf{u}}\otimes\nabla-\nabla\otimes\dot{\mathbf{u}}) \end{cases} \tag{5}$$

with $\otimes$ being the dyadic product symbol.

2.2. Constitutive equations

Under rainfall circumstance the slope soil lies in an unsaturated state and experiences a process of saturation. The elasto-plastic constitutive relationship proposed by Alonso et al. [30] for the unsaturated soil with infinitesimal deformation has been shown to be very effective and is one of the most frequently used constitutive law. Herein this constitutive law is generalized to large deformation range by employing the classical rate formulation where the Jaumann's objective stress rate is used to incorporate the plastic and damage behavior

$$\boldsymbol{\sigma}^{*\diamond}=\mathbf{C}^{epd}:(\dot{\boldsymbol{\varepsilon}}-\mathbf{1}\dot{\varepsilon}_s) \tag{6}$$

where $\mathbf{C}^{epd}$ denotes the elasto-plastic damage tensor, $\mathbf{1}$ is the second order identity tensor, $\dot{\varepsilon}_s$ is the suction-induced volumetric rate of deformation tensor, respectively. $\boldsymbol{\sigma}^{*\diamond}$ is the Jaumann's objective rate of the net stress $\boldsymbol{\sigma}^*$

$$\boldsymbol{\sigma}^{*\diamond}=\dot{\boldsymbol{\sigma}}^*-\boldsymbol{\omega}\boldsymbol{\sigma}^*+\boldsymbol{\sigma}^*\boldsymbol{\omega} \tag{7}$$

The suction-induced volumetric rate of deformation tensor can be related to the change of suction as [30]

$$\dot{\varepsilon}_s=D_s^{-1}\dot{s} \tag{8}$$

where D_s is a material parameter.

To describe the material separation an isotropic strain-based damage model [31,32] is employed in this study. The scalar damage