Engineering Analysis with Boundary Elements 42 (2014) 67–76

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

The Isoparametric Reproducing Kernel Particle Method for nonlinear deformation of plates

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ARTICLE INFO

Article history:
Received 7 April 2013
Accepted 25 August 2013
Available online 17 September 2013

Keywords:
Meshfree
Reproducing Kernel Particle Method
Nonlinear
Plates and shells
Numerical integration

ABSTRACT

This paper proposed the new Isoparametric Reproducing Kernel Particle Method (IsoRKPM) for modeling nonlinear plate and shell deformation problems. The Reproducing Kernel shape functions are constructed on a two-dimensional parent domain. Following the concept of isoparametric mapping, the RK shape functions are directly used to approximate the plate geometry. A High Order Nodal Integration (HONI) is developed to integrate the Galerkin weak form of the Mindlin plate equilibrium equations. The proposed IsoRKPM with HONI is applied to solve several benchmark problems. Both modal and convergence analyses show that HONI provides more stable and accurate solutions than the nodal integration method.

1. Introduction

Finite Element Method (FEM) is a well developed method for analyzing engineering problems involving structural members, such as plates and shells [1,2]. Recent focus of these researches are directed to the geometric exactness [3,4] and hybrid numerical integration methods [5]. Although the FEM has been efficiently used to model finite deformation problems with material and geometric non-linearity, it suffers mesh distortion problem when large deformation is encountered [6]. Although some advanced FEM algorithms allow re-meshing during computation [7], the mesh distortion and mesh dependency still affect the behavior of the structure.

The development of meshfree methods started from the Smooth Particle Hydrodynamics (SPH) method [8,9], which was first used for the modeling of astronomical problem. The SPH formulation is later extended to the field of solid and fluid dynamics [10,11]. Another group of meshfree methods, including the Element Free Galerkin Method (EFG) [12] and Reproducing Kernel Particle Method (RKPM) [13] are developed to improve the accuracy of the original SPH approximation. All the meshfree methods have the characteristic of constructing the approximation without the need of a mesh. Therefore, the meshfree methods can be more suitable for dealing with problems with complex geometry and large deformations. The meshfree method is also employed to plate and shell problems. Krysl and Belytschko used shape function with quadratic basis and larger domains of influence to prevent the membrane locking [14]. In 2000, Li et al. introduced a highly smooth 3-D RK shape function for plate analysis, which mitigates the locking phenomenon [15]. Also, Li et al. [16] and Soric et al. [17] applied different orders of interpolation functions for in-plane and transverse displacement to eliminate the thickness locking. Alternatively, the locking phenomenon can also be suppressed by using an appropriate integration technique, such as stabilized conforming nodal integration [18,19]. Another issue of the meshfree method is the lack of Kronecker delta property of meshfree shape functions, which leads to the difficulty of applying the essential boundary conditions. Chen et al. [20,21] introduced a transformation technique to implement the essential boundary condition. In 2008, Wang and Chen proposed a Hermite Reproducing Kernel Particle Method (HRKPM) [22–24] for Kirchhoff plates to impose the Kirchhoff Mode Reproducing Condition (KMRC), and the HRKPM is also applied for the buckling analysis of thin Kirchhoff plates in 2012 [25].

When modeling a shell structure with a complex geometry, the general RKPM has its inherent drawback as the plates or shells are co-plane with the basis of RK approximation. This problem will lead to a singular system in the construction of RK shape function. In 2006, Chen and Wang [26] proposed two methods to avoid the singularity of the moment matrix in the three-dimensional RK shape function. The first method is the dummy nodes technique, which uses the physical and dummy nodes on the shell surface to formulate the RK shape function. The second method is the pseudo-inverse method, which requires solving the eigenvalues of the moment matrix. In 2000, Noguchi et al. [27] introduced a parametric mapping technique, which projects a two-dimensional surface discretized by Element Free Galerkin (EFG) on to the three-dimensional middle surface of the shell by using the Lagrange

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0955-7997/$ - see front matter
http://dx.doi.org/10.1016/j.enganabound.2013.08.013

polynomial functions. Under this framework, the sample points, which are used to construct the Lagrange polynomial functions, can be different from the discretization points used to build the EFG shape functions. This method is similar to FEM when an isoparametric element is used.

In this study, we use this concept to construct RK approximation. First, we build a two-dimensional space to construct the Reproducing Kernel shape function, which is used to approximate the state and field variables of the Mindlin plate formulation. The geometry of the shell can then be built in this two-dimensional coordinate system by using the same discretization points of the shape function to perform the RK approximation for the surface. A transformation method is introduced to remove the error of the geometry due to the lack of Kronecker delta property of the RK shape functions. This method ensures the continuity of RK approximation and flexibility to change the order of approximation which makes the method efficient for modeling complex geometry and discontinuities.

The paper is organized as follows. Section 2 introduces the basic formulation of the Reproducing Kernel Particle Method and the Mindlin plate theory. In Section 3, the isoparametric RK approximation is constructed to define the surface. In Section 4, the equilibrium based on the local coordinate is introduced. Then the Galerkin-based discrete equations with RKPM discretization are derived. In the end of Section 4, a High Order Nodal Integration method is applied to the discrete equations. In Section 5, several numerical examples for linear and nonlinear analysis are presented to demonstrate the accuracy and robustness of the proposed method. Finally the conclusions are given in Section 6.

2. Basic formulation

2.1. General Reproducing Kernel Particle Method

The function $u(x)$ approximated by RK approximation can be presented as a linear combination of the RK shape functions $\Phi_a(\boldsymbol{x};\boldsymbol{x}-\boldsymbol{x}_I)$ and the associated nodal coefficients u_I, which is

$$u^h(\boldsymbol{x}) = \sum_{I=1}^{NP} \Phi_a(\boldsymbol{x};\boldsymbol{x}-\boldsymbol{x}_I)u_I \tag{1}$$

where $u^h(x)$ is the approximation of function $u(x)$, NP is the number of RK points in the domain Ω and x_I is the coordinate of I-th particle. The RK shape functions $\Phi_a(x;x-x_I)$ are defined as

$$\Phi_a(\boldsymbol{x};\boldsymbol{x}-\boldsymbol{x}_I) = C(\boldsymbol{x};\boldsymbol{x}-\boldsymbol{x}_I)\phi_a(\boldsymbol{x}-\boldsymbol{x}_I) \tag{2}$$

where ϕ_a is the kernel function with support size a, and $C(\boldsymbol{x};\boldsymbol{x}-\boldsymbol{x}_I)$ is the correction function [13], which is an N-th order polynomial function

$$C(\boldsymbol{x};\boldsymbol{x}-\boldsymbol{x}_I) = \sum_{i+j+k=0}^{N} b_{ijk}(\boldsymbol{x})(x-x_I)^i(y-y_I)^j(z-z_I)^k = \mathbf{H}^T(\boldsymbol{x}-\boldsymbol{x}_I)\mathbf{b}(\boldsymbol{x}) \tag{3}$$

and $\mathbf{H}(\boldsymbol{x}-\boldsymbol{x}_I)$ and $\mathbf{b}(\boldsymbol{x})$ are the polynomial vector and the associated unknown coefficient vector respectively. Introducing the N-th order consistency condition,

$$\sum_{I=1}^{NP} \Phi_I(\boldsymbol{x})x_I^i y_I^j z_I^k = x^i y^j z^k \quad 0 \le i+j+k \le N \tag{4}$$

then the coefficient vector can be calculated by solving the following local equation:

$$\mathbf{b}(\boldsymbol{x}) = \mathbf{M}^{-1}(\boldsymbol{x})\mathbf{H}(0) \tag{5}$$

where

$$\mathbf{H}^T(0) = [1, 0, \ldots, 0] \tag{6}$$

and

$$\mathbf{M}(\boldsymbol{x}) = \sum_{J=1}^{NP} \mathbf{H}(\boldsymbol{x}-\boldsymbol{x}_J)\mathbf{H}^T(\boldsymbol{x}-\boldsymbol{x}_J)\phi_a(\boldsymbol{x}-\boldsymbol{x}_J) \tag{7}$$

Substituting Eq. (5) into (3), and Eq. (2) can be presented as

$$\Phi_I(\boldsymbol{x};\boldsymbol{x}-\boldsymbol{x}_I) = \mathbf{H}^T(0)\mathbf{M}^{-1}(\boldsymbol{x})\mathbf{H}(\boldsymbol{x}-\boldsymbol{x}_I)\phi_a(\boldsymbol{x}-\boldsymbol{x}_I) \tag{8}$$

In this paper, a cubic B-spline function will be applied as the kernel function.

$$\phi_a(r) = \begin{cases} \frac{2}{3} - 4r^2 + 4r^3 & \text{for} \quad 0 \le |r| \le 0.5 \\ \frac{4}{3} - 4r + 4r^2 - \frac{4}{3}r^3 & \text{for} \quad 0.5 \le |r| \le 1 \\ 0 & \text{for} \quad |r| > 1 \end{cases} \tag{9}$$

where $r = (\boldsymbol{x}-\boldsymbol{x}_I)/a$.

2.2. Mindlin plate theory

The Mindlin plate theory [28] assumes that the plane normal to the middle surface of the plate before deformation will remain straight but will not remain normal to the middle surface, which is shown in Fig. 1.

The transverse shear strain of the plate is considered by introducing two additional degrees of freedom, rotational parameters θ_x and θ_y. In order to construct the local equilibrium equations, the local coordinate system x, y, z is constructed based on the tangent and normal directions of the middle surface of the plate, which is shown in Fig. 2. The displacement in the local coordinate system can be expressed as

$$\mathbf{u} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} u_0 \\ v_0 \\ w_0 \end{bmatrix} + z\begin{bmatrix} \theta_y \\ -\theta_x \\ 0 \end{bmatrix} \tag{10}$$

where the vector $[u\ v\ w]^T$ are the displacements in e_1', e_2', e_3' direction, and the index "$_0$" denotes the variables measured at the middle surface of the plate. The two variables θ_x and θ_y are the rotations about x-, and y-axes respectively.

In Mindlin plate, the strain normal to the local x–y surface is neglected. Therefore the general strain vector in local coordinates

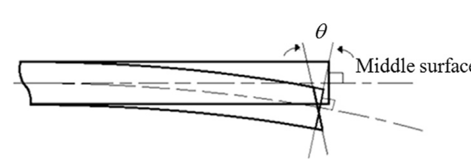

Fig. 1. Cross plane assumption.

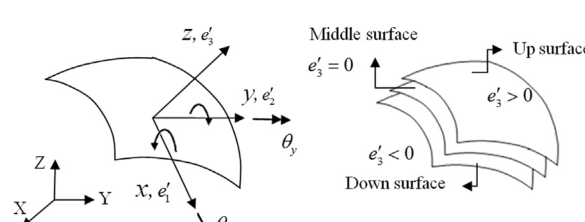

Fig. 2. The local coordinate system based on the tangent and normal directions of the middle surface of the plate.