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Reconstruction of dynamically changing boundary of multilayer heat conduction composite walls ☆

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ABSTRACT

This paper presents an inverse reconstruction procedure to determine the inner boundary location of heat conduction composite walls from the measurement data of temperature and heat flux on the exterior boundary. Our procedure uses a meshless forward solver that was developed for solving inhomogeneous heat transfer problems across a multilayer composite wall with Cauchy conditions. The forward solver uses the radial basis functions (RBFs) approximation in both time and space in a unified fashion, and hence is well suited for inverse problems. In this work, we consider that the length of the inner layer of the composite wall may vary caused by the material erosion at very high temperature such as in iron-making blast furnaces. In order to mitigate the ill-posed inverse problem, we use the Tikhonov regularization technique to obtain a stable and accurate numerical approximation of the moving boundary. Numerical experiments for a number of examples are presented to demonstrate the effectiveness of our inverse procedure. It can be observed that the error of the inverse solution is smaller or at the same level of noises in the simulated measurement data, demonstrating that our inverse procedure is effective and stable with respect to noisy data for moving boundary problems.

1. Introduction

In this paper, we consider the determination of an erosion boundary in an inhomogeneous heat transfer problem with Cauchy condition.

Erosion of material is often encountered in engineering systems that involve operations at very high temperature such as iron-making blasts. Iron-making blast furnaces are usually made of composite material which consists of more than one material with properties of high temperature resistance, heat insulation and a high strength. During the production processes, the inner layer is in direct contact with the melted iron, and is subject to constant erosion due to the extreme hashing conditions, leading to a change in the thickness of the inner layer. It is therefore, important to monitor the eroded thickness of the accreted refractory wall in order to avoid molten metal breaking out. Because of the inaccessibility to the inner surface of the furnaces, we need to somehow reconstruct it through an inverse procedure, which is a very important topic in the nondestructive evaluations [1]. In such an inverse procedure, we can only depend on the time-history of the temperature and heat flux data on the outer surface of the container to reconstruct the dynamic changes of the inner boundary. Such an inverse boundary problem belongs to a family of problems that have inherited the property of being ill-posed in the Hadamard sense [2].

In recent years, the boundary identification has already been researched extensively [3–19]. To perform inverse analyses, one needs an efficient forward solver. Finite difference methods (FDM) and finite element methods (FEM) have been used as a forward solver to reconstruct the erosion of boundary [3–6]. When large numbers of grids in these methods are required for problems with moving boundary, mesh distortion may occur [20]. Hence various types of efficient meshfree methods have been recently presented [22–33]. As the presentation of meshfree collocation methods [20,21], many meshfree methods have been proposed to solve the boundary identification problem. The method of fundamental solution (MFS) has been adopted to deal with this problem [11–16]. The results in these references are effective and stable for homogeneous equations. The least-squares collocation meshless method

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[7], radial point interpolation method [8] and radial basis function collocation method [9] have been presented to solve this reconstruction problem.

In this work, we aim to device an inverse reconstruction procedure to determine the inner boundary location of heat conduction composite walls from the measurement date of temperature and heat flux on the exterior boundary, known as Cauchy conditions. Our procedure uses a meshless forward solver for inhomogeneous heat transfer problems across a multilayer composite wall. The forward solver uses the radial basis functions (RBFs) approximation in both time and space in a unified fashion, and hence it is well suited for inverse problems. In this work, we consider that the length of the inner layer of the composite wall may vary caused by material erosion at very high temperature such as in iron-making blast furnaces. First, we consider that the heat transfer across a composite wall is assumed to be infinitely long in length direction and hence the heat source and any heat exchanges are also independent of length [34]. We assume that the heat conducts only through the composite wall and thus it is simplified to a one-dimensional problem in space. We then propose unified space–time boundary identification method based on RBFs, which treat time as a dimension in the same way as the spatial coordinate. Substituting the governing equations, Cauchy conditions and the interface conditions, we treat three layers as a whole to set a single set of linear algebraic system equations to avoid the accumulation of numerical error due to layer-by-layer recursion process [15]. To overcome the ill-posedness, we use the discrete Tikhonov regularization technique to regularize the ill-conditioned linear system of equations.

The paper is organized as follows. In Section 2, we introduce the setting of a boundary identification problem. Section 3 presents the unified space–time method. Section 4 introduces the inverse procedure. Five numerical experiments are conducted to examine the accuracy and stability of our proposed method in Section 5. Finally, in Section 6 we give conclusions.

2. The setting of the problem

The space–time domain of a boundary identification problem in a three-layer composite wall is shown schematically in Fig. 1. We consider a typical heat transfer inverse problem with a set of Cauchy boundary conditions (both the temperature and flux are all specified) on the known boundary l_1. The moving boundary function (in terms of both space and time) is to be determined.

The governing equations for the temperature distribution in the inhomogeneous heat transfer media with three layers are governed by the following equations [34,35]:

$$\frac{\partial u_1(x,t)}{\partial t} = a_1 \frac{\partial^2 u_1(x,t)}{\partial x^2} + f_1(x) \quad \text{in } D_1 \tag{1}$$

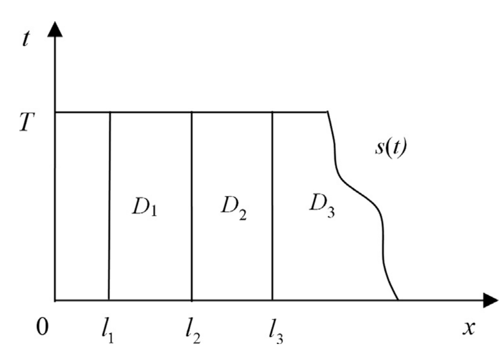

Fig. 1. The space–time domain.

$$\frac{\partial u_2(x,t)}{\partial t} = a_2 \frac{\partial^2 u_2(x,t)}{\partial x^2} + f_2(x) \quad \text{in } D_2 \tag{2}$$

$$\frac{\partial u_3(x,t)}{\partial t} = a_3 \frac{\partial^2 u_3(x,t)}{\partial x^2} + f_3(x) \quad \text{in } D_3 \tag{3}$$

where the temperature field in each layer is denoted by function $u_i(x,t)$ $(i=1,2,3)$, T represents the maximum time of interest for the time evolution of our inverse problem, $f_1(x)$, $f_2(x)$ and $f_3(x)$ are the heat source in each of the three layers. $D_1=[l_1,l_2]\times[0,T]$, $D_2=[l_2,l_3]\times[0,T]$, $D_3=[l_3,s(t)]\times[0,T]$ are the space–time subdomains. The thermal diffusion coefficient a_i $(i=1,2,3)$ in each layer can be calculated using the equation $a_i=k_i/\rho_i c_i$, where k_i is the thermal conductivity coefficient, ρ_i is the density and c_i is the specific heat capacity in each layer.

The Cauchy conditions are assumed to be given on l_1:

$$u_1(l_1,t) = \overline{u}_1(l_1,t), \quad 0 \le t \le T \tag{4}$$

$$\frac{\partial u_1(l_1,t)}{\partial x} = \overline{q}_1(l_1,t), \quad 0 \le t \le T \tag{5}$$

and continuity conditions in the interfaces are

$$u_1(l_2,t) = u_2(l_2,t), \quad 0 \le t \le T \tag{6}$$

$$k_1 \frac{\partial u_1(l_2,t)}{\partial x} = k_2 \frac{\partial u_2(l_2,t)}{\partial x}, \quad 0 \le t \le T \tag{7}$$

$$u_2(l_3,t) = u_3(l_3,t), \quad 0 \le t \le T \tag{8}$$

$$k_2 \frac{\partial u_2(l_3,t)}{\partial x} = k_3 \frac{\partial u_3(l_3,t)}{\partial x}, \quad 0 \le t \le T \tag{9}$$

Reconstruction of moving boundary is to determine the moving boundary function $s(t)$ from a condition

$$u_3(s(t),t) = u_s(t) \tag{10}$$

where $u_s(t)$ is a given function or a constant that is the melting point of the medium in the inner layer 3.

Because the Cauchy problems are well known for highly ill-posedness, a proper regularization is required for stable and reliable solution.

3. The unified space–time method

For unsteady state heat transfer problems, the temperature fields are the function of the position and the time. In our unified space*-time approximation for the temperature field, we treat the time as a dimension similar to the space. Following the idea of radial basis functions [21,36,37], we assume that an approximation to the temperature solution for each layer can be expressed as follows:

$$\hat{u}_1(x,t) = \sum_{j=1}^{N_1} \alpha_{1j}\varphi_{1j}(x,t), \quad (x,t) \in D_1 \tag{11}$$

$$\hat{u}_2(x,t) = \sum_{j=1}^{N_2} \alpha_{2j}\varphi_{2j}(x,t), \quad (x,t) \in D_2 \tag{12}$$

$$\hat{u}_3(x,t) = \sum_{j=1}^{N_3} \alpha_{3j}\varphi_{3j}(x,t), \quad (x,t) \in D_3 \tag{13}$$

where $\varphi_{ij}(x,t)$ $(i=1,2,3)$ is the RBF function and $\alpha_{ij}(x,t)$ $(i=1,2,3)$ is the coefficient. We set some interior points in each space–time domain for each layer for collocation purposes so that the coefficient $\alpha_{ij}(x,t)$ can be determined. In our scheme, we use the Kansa method [38,39]:

$$\varphi_i = \varphi_i(r) \tag{14}$$