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Minmax weighted earliness-tardiness with identical processing times and two competing agents

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ABSTRACT

A classical single machine scheduling problem is that of minimizing the maximum weighted deviation of the job completion times from a common due-date, assuming identical processing times. We extend this problem to a setting of two competing agents sharing the same machine. We first focus on the case that the objective is of minimizing the maximum weighted deviation of the jobs of the first agent subject to an upper bound on the maximum weighted deviation of the jobs of the second agent. Then we extend this model to a setting of asymmetric cost structure, i.e., the (job- and agent-dependent) earliness and tardiness costs may be different. We also consider a modified model with a minsum measure for the second agent: the objective is of minimizing the maximum weighted deviation of the jobs of the first agent from a common due-date subject to an upper bound on the total weighted deviation of the jobs of the second agent. All these models are also extended to a general setting of job-dependent due-dates. Polynomial time solutions are introduced for all the problems studied in this paper.

1. Introduction

Mosheiov and Shadmon (2001) studied a single machine Just-In-Time scheduling problem, where all the jobs share a common due-date, job processing times are assumed to be identical, and the objective is minimizing the maximum weighted deviation of the job completion times from the due-date. They showed that the problem can be reduced to a special structured linear program which is solved in linear time in the number of jobs (n). The total running time required for solving the problem is, however, $O(nlogn)$ due to the initial sorting of the jobs.

In this paper we extend this problem to a setting of *two competing agents*. In this class of scheduling problems, which attracted an increasing number of researchers in the last decade, each one of two agents (schedulers, producers) has his own set of jobs and his own objective function (a scheduling measure). The agents share a common processor (machine). In most papers dealing with this setting, the goal is to optimize the measure of one agent subject to an upper bound on the measure of the second agent. The first two papers in this new line of research are those of Baker and Smith (2003) who studied basic scheduling criteria such as makespan, maximum lateness and total weighted completion time, and of Agnetis, Mirchandani, Pacciarelli, and Pacifici (2004) who focused on general regular (job-dependent) measures, considered additional objective functions (such as number of tardy jobs), and extended some of the results to shops. Many extensions of these models to various combinations of scheduling measures and machine settings have been published since then, among them: Cheng, Ng, and Yuan (2006, 2008), Ng, Cheng, and Yuan (2006), Agnetis, Pacciarelli, and Pacifici (2007), Lee, Choi, Leung, and Pinedo (2009), Liu, Tang, and Zhou (2009), Liu, Zhou, and Tang (2010), Mor and Mosheiov (2010, 2011, 2014), Leung, Pinedo, and Wan (2010), Cheng, Cheng, Wu, Hsu, and Wu (2011), Gawiejnowicz, Lee, Lin, and Wu (2011), Wu, Huang, and Lee (2011), Li and Yuan (2012), Li and Hsu (2012), Cheng, Liu, and Lee (2014), Shiau, Tsai, Lee, and Cheng (2015), Dover and Shabtay (2015), Yin, Cheng, Wan, Wu, and Liu (2015a), Yin, Cheng, Yang, and Wu (2015b), Choi (2015), Yin, Cheng, Cheng, Wang, and Wu (2016a), Yin, Wang, Cheng, Wang, and Wu (2016b), Yin, Wang, Wu, and Cheng (2016c), Wu et al. (2017), Yin, Cheng, Wang, and Wu (2017). The recent book of Agnetis, Billaut, Gawiejnowicz, Pacciarelli, and Soukhal (2014) summarizes all the published results on scheduling with competing agents.

A relatively small number of papers focused on settings of two competing agents in the context of *Just-In-Time* (i.e., when both earliness and tardiness costs are considered). To our knowledge, the list of the relevant published papers contains the following: Gerstl and Mosheiov (2013) studied a setting of identical jobs

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where the objective is minimum total weighted deviation of the job completion times of one agent subject to an upper bound on the maximum weighted deviation of the job completion times of the second agent. They introduced polynomial time solutions even for multi-machine (parallel identical and uniform) settings. Polyakovskiy and M'Hallah (2014) studied multi-agent settings with different types of agents, job-dependent processing times and due-dates. They introduced and tested a heuristic approach to a general setting of unrelated machines. Gerstl and Mosheiov (2014) focused on a single machine, a common (possibly restrictive) due-date and job-dependent processing times. The goal of one agent is of a minsum type and that of the second agent is of a minmax type. A pseudo-polynomial dynamic programming was introduced for the two-agent case, and an efficient heuristic was tested for the (strongly NP-hard) case of multi-agents. Wang et al. (2015) studied a setting where the due-dates are given variables that must be assigned to individual jobs. They proved that the problem of minimizing total weighted earliness-tardiness of one agent subject to an upper bound on a scheduling measure of the second agent (for a number of relevant measures) is strongly NP-hard. Wang, Yin, Cheng, Cheng, and Wu (2016) studied a problem combining two-agent scheduling and due-date assignment. The objective was a linear combination of total weighted earliness and number of tardy jobs. They introduced a polynomial time solution for the objective of minimizing a linear combination of the measures of both agents, and proved NP-hardness for the constrained setting (minimizing the measure of one agent subject to an upper bound on the measure of the other). Yin, Wang, Wu, and Cheng (2016c) studied due-date assignment problems with two competing agents, i.e., in their setting the due-dates are decision variables. Specifically, they considered the CON and SLK models, where the objective of the first agent is minimizing due-date cost plus the weighted number of tardy jobs. A number of minmax and minsum measures were assumed for the second agent. Pseudo polynomial dynamic programs and a fully polynomial approximation scheme were introduced. Two recently published papers focused on minimizing the weighted number of Just-in-Time jobs in the context of competing agents. Yin, Cheng, Cheng, Wang, and Wu (2016a) studied these measures on unrelated parallel machines, and Yin et al. (2017) considered a flowshop. Both papers introduce pseudo-polynomial dynamic programs and fully polynomial approximation schemes.

An important special case in scheduling theory is that of identical jobs. Beyond its theoretical importance, this setting reflects many real-life situations, e.g., that of production lines of identical items. Numerous papers have been published on scheduling of identical jobs. In the context of two competing agents, we refer the reader to the recent paper of Oron, Shabtay, and Steiner (2015) who introduced polynomial time solutions for a large set of problems, and proved NP-hardness for many others. A key assumption in the models studied by Oron et al. (2015) is that the measures considered (for both agents) are regular, i.e., they are non-decreasing functions of the job completion times.

In this paper we study a single machine scheduling problem with (i) two-competing agents, (ii) identical jobs, and (iii) Just-In-Time measures. Specifically, we consider a minmax scheduling measure for one agent, and either a minmax or a minsum measure for the second agent. We first extend the original single agent setting studied by Mosheiov and Shadmon (2001) to that of minimizing the maximum weighted deviation of the job completion times of the first agent from a common due-date subject to an upper bound on the maximum weighted deviation of the jobs of the second agent. Then we further extend this model to a setting of asymmetric cost structure, i.e., each job of both agents has two (not necessarily identical) weights for earliness and tardiness. We also consider a modified model with a minsum measure for the second agent: the objective is of minimizing the maximum weighted deviation of the job completion times of the first agent from a common due-date subject to an upper bound on the *total* weighted deviation of the jobs of the second agent. Following Mosheiov and Shadmon (2001), all the above models consider a common due-date for all the jobs. In the last part of the paper we extend all these three models to a setting of *job-dependent due-dates*. We introduce polynomial time solutions for all the problems studied in this paper.

The paper is organized as follows. Section 2 contains the notation and the formulation of the problems. Section 3 studies the first extension to a two-agent setting. Section 4 introduces the solution for the extension to an asymmetric cost structure. Section 5 focuses on the minsum measure for the second agent. In Section 6 we provide the solutions for the extensions of all these problems to the setting of job-dependent due-dates.

2. Formulation

We study a single machine two-agent scheduling problem. Agent X processes n^X jobs, and agent Y processes n^Y jobs $(n = n^X + n^Y)$. We assume that all the jobs of both agents have identical processing times, denoted by p. However, each job has its own weight, denoted by $w_j^Z,\ j = 1,\dots,n^Z,\ Z = X,\ Y$. All the jobs share a common due-date, denote by d. All data are assumed to be integers. We assume that d is sufficiently small to force the first scheduled job to start at time zero (also known in scheduling literature a "restrictive due-date"). All jobs are available at time zero, and preemption is not allowed.

For a given schedule, C_j^Z denotes the completion time of job j of agent Z, and $|C_j^Z - d|$ denotes the absolute deviation of the completion time of job j from the common due-date, $j = 1,\dots,n^Z,\ Z = X, Y$. As mentioned above, our objective is to minimize the cost of one agent (X), subject to an upper bound on the measure of the second agent (Y). Following Mosheiov and Shadmon (2001) who studied the single agent case, we consider a minmax measure for both agents X and Y. Specifically, the objective function of agent X is minimizing the Maximum Weighted Absolute Deviation from the common due-date among all the jobs: $MWAD(X) = \max_{j=1,\dots,n^X}\{w_j^X|C_j^X - d|\}$. Similarly, the minmax measure for agent Y is: $MWAD(Y) = \max_{j=1,\dots,n^Y}\{w_j^Y|C_j^Y - d|\}$. If Q denotes the upper bound on the measure of agent Y, then the problem studied here is:

$$\mathbf{P1}: 1/p_j^Z = p,\ d_j^Z = d,\ w_j^Z/MWAD(X) : MWAD(Y) \leqslant Q.$$

As mentioned, several modifications of problem **P1** are considered in this paper. The first extension assumes asymmetric cost structure. Thus, the earliness and tardiness weights of job j of agent Z are denoted, respectively, by α_j^Z, and $\beta_j^Z,\ j = 1,\dots,n^Z,\ Z = X, Y$. Let $E_j^Z = \max\{d - C_j^Z, 0\}$, and $T_j^Z = \max\{C_j^Z - d, 0\}$ denote the earliness and tardiness, respectively, of job j of agent Z, $j = 1,\dots,n^Z,\ Z = X,\ Y$. The relevant measures become:

$$MWAD(Z) = \max_{j=1,\dots,n^Z}\{\alpha_j^Z E_j^Z + \beta_j^Z T_j^Z\},\ Z = X,\ Y. \text{ Thus,}$$

$$\mathbf{P2}: 1/p_j^Z = p, d_j^Z = d, \alpha_j^Z, \beta_j^Z/MWAD(X) : MWAD(Y) \leqslant Q.$$

Problem **P3** consists of the following modification of problem **P1**: the scheduling measure of agent ***Y*** is of a minsum type. Specifically, we consider Total Weighted Absolute Deviations of job completion times from the common due-date: $TWAD(Y) = \sum_{j=1}^{n^Y}\{w_j^Y|C_j^Y - d|\}$. Problem **P3** is therefore: