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A chance constrained programming approach for uncertain *p*-hub center location problem

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ABSTRACT

The *p*-hub center location problem aims to locate *p* hubs and allocate other nodes to these hub nodes in order to minimize the maximal travel time. It is more important for time-sensitive distribution systems. Due to the presence of uncertainty, more researches are recently focused on the problem in non-deterministic environment. This paper joins the research stream by considering travel times as uncertain variables instead of random variables or fuzzy ones. The goal is to model the *p*-hub center problem based on experts' subjective belief in the case of lack of data. The uncertain distribution of the maximal travel time is first derived and then a chance constrained programming model is formulated. The deterministic equivalent forms are further given when the information of uncertainty distributions is provided. A hybrid intelligent algorithm is designed to solve the proposed models and numerical examples are presented to illustrate the application of this approach and the effectiveness of the algorithm.

1. Introduction

Hubs are, in practice, commonly used in transportation, logistics, telecommunication systems and serves as consolidation, switching and sorting centers in a complex system (Campbell, Lowe, & Zhang, 2007). The *p*-hub center location problem involves the location of hub facilities and the routing from origins to destinations to minimize the maximal travel time (or cost, distance, etc.) between any origin-destination pair. The problem is useful for time-sensitive distribution systems such as emergency services, express mail service or timely delivery of perishable products (Campbell et al., 2007). Therefore, since it was initialized by Campbell (1994), more research interests are draw to study and extend the problem. The latest review can be found in Campbell and O'Kelly (2012) and Farahani, Hekmatfar, Arabani, and Nikbakhsh (2013).

In general, hub location problems are strategic in nature, which implies that the travel times may change with time. Therefore, it is meaningful to consider the problem within an uncertain environment. One main stream of research is to deal with uncertainty as randomness, i.e., stochastic *p*-hub center location problem. Sim, Lowe, and Thomas (2009) was first one to present stochastic *p*-hub center problem and established a chance-constrained programming with service-level constraints. Then Yang, Liu, and Zhang (2011) extended the problem by assuming discrete random travel time, and Hult, Jiang, and Ralph (2014) developed exact solution approaches based on variable reduction and a separation algorithm to solve an uncapacitated single allocation case.

Another stream of research is to study *p*-hub center location problem in fuzzy environment. For instance, Yang, Liu, and Yang (2013a) first proposed a fuzzy *p*-hub center problem in which the travel times are characterized by normal fuzzy vectors. Based on the same setting, Yang, Liu, and Yang (2013b) further presented a risk aversion formulation by adopting value-at-risk criterion in the objective function. By using the criterion, Yang, Liu, and Yang (2014) recently developed a robust optimization method to describe travel times by employing parametric possibility distributions. Similarly, Bashiri, Mirzaei, and Randall (2013) considered a hybrid approach to the capacitated case with fuzzy data and employed genetic algorithm to solve the problem.

In practice, there are often lack of data about future changes or the implementation will take considerable time or cost. A more feasible and economic way is to estimate the parameters by experts based on their subjective information and experiences. Liu (2010) proposed uncertainty theory to describe such a nondeterministic phenomena. Since then, uncertainty theory has been applied to practical problems such as Chen, Kar, and Ralescu (2012) and Gao, Yang, Li, and Kar (2015) as a new approach dealing with uncertainty. Gao (2011) introduced the shortest path problem

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and gave the uncertainty distribution of the shortest path length, and Gao (2012) proposed single facility location model in which products assignment is evaluated by the concept of satisfaction degree. Wen, Qin, and Kang (2014) presented chance-constrained formulation for capacitated facility location-allocation problem when demands are uncertain. Han, Peng, and Wang (2014) studied the maximum flow in an uncertain network in which arc capacities are uncertain variables.

This paper focuses on the single allocation p-hub center location problem with lack of data about travel times. Instead of estimating the travel times by statistical methods, this paper assume that these quantities are evaluated by domain experts. Specifically, in the framework of uncertainty theory, this paper regards the travel times between the origins and destinations as uncertain variables and propose a new formulation for the problem. Chance constrained programming approach is employed to model the problem and further formulate an uncertain optimization model for decision makers. The proposed model is then transformed to deterministic equivalent forms when uncertainty distributions are provided. In order to solve these models, we design a hybrid intelligent algorithm by combining principle of nearby into genetic algorithm.

The rest of the paper is organized as follows. In Section 2, we first formulate a chance constrained programming model and then discuss its deterministic equivalent forms. Section 3 provides a heuristic solution procedure to solve the proposed models. In Section 4, numerical examples are given to illustrate the application and effectiveness of the proposed models. A brief conclusion is given in Section 5 and finally some necessary preliminaries are given in Appendix A.

2. Uncertain p-hub center problem

This section is divided into three subsections. The first subsection describes the p-hub center location problem in uncertain environment, the second one formulates a chance constrained programming model, and the third one discusses the deterministic equivalent forms of the proposed model.

2.1. Problem description

Assume that there are n nodes in the network and the number of hubs to locate is given exogenously and denoted by p. Let η_{ij} be the travel time on the link from node i to j, which is considered as an uncertain variable defined on the uncertainty space $(\Theta, \mathcal{P}, \mathcal{M})$. A path $i \to k \to m \to j$ represents a unit of demand originating at node i destined for j traveling through hub k first then hub m. If α is a discount factor denoting economies of scale on the inter-hub linkage, then the total travel time on this path is $\eta_{ik} + \alpha\eta_{km} + \eta_{mj}$ which is also an uncertain variable. Note that $k = m$ implies that only one hub is used and naturally the discount vanishes.

The decision variable x_{ik} is introduced as a binary variable to represent the assignment of node i to hub k for $i, k = 1, 2, \ldots, n$. Here $x_{kk} = 1$ indicates that node k is assigned to itself and it is actually a hub node. We assume that all of hub nodes are connected to one another, however, any two non-hub nodes are never connected directly. Moreover, each non-hub node is assigned to a single hub. In addition, there is no cost for setting up hubs and there are no capacity limits.

If all the travel time η_{ij} are all known in advance, i.e., deterministic values, the uncertain p-hub center location problem becomes to a traditional deterministic one, the aim of which is to minimize the maximal travel time. We denote the travel time by w_{ij} in this part. Typically, the deterministic p-hub center problem can be formulated as

$$\begin{cases} \min\limits_{\boldsymbol{x}} \quad f(\boldsymbol{w},\boldsymbol{x}) = \max\limits_{i,j,k,m}\{(w_{ik} + \alpha w_{km} + w_{mj})x_{ik}x_{jm}\} \\ \text{s.t.} \quad \sum_{k=1}^{n} x_{kk} = p \qquad \text{(i)} \\ \qquad \sum_{k=1}^{n} x_{ik} = 1, \quad i = 1,2,\ldots,n \qquad \text{(ii)} \\ \qquad x_{ik} \leqslant x_{kk}, \quad i,k = 1,2,\ldots,n \qquad \text{(iii)} \\ \qquad x_{ik} = \{0,1\}, \quad i,k = 1,2,\ldots,n \qquad \text{(iv)} \end{cases} \tag{1}$$

where $\boldsymbol{w} = (w_{ij})$ and $\boldsymbol{x} = (x_{ij}),\ i,j = 1,\ldots,n$.

For model (1), the objective function $f(\boldsymbol{w},\boldsymbol{x})$ represents the maximal travel time between two nodes in the p-hub location $\boldsymbol{x}$. Constraint (i) stipulates that exactly p hub nodes are chosen and constraints (ii) stipulate that non-hub node i is assigned to precisely one hub node. Constraints (iii) enforce that node i is assigned to a hub node at k only if a hub is located at node k. Finally, constraints (iv) define the decision variable types. For sake of simplicity, we write

$$X = \{\boldsymbol{x} | \boldsymbol{x} \text{ satisfies constraints (i), (ii), (iii) and (iv) of model (1)}\},$$

which is the collection of all feasible solutions of p-hub center location problem.

Denote by $\boldsymbol{x}^*$ the optimal solution of model (1). Then the optimal objective value of model (1) can be expressed as follows

$$F(\boldsymbol{w}) = f(\boldsymbol{w},\boldsymbol{x}^*) = \min_{\boldsymbol{x}\in X} f(\boldsymbol{w},\boldsymbol{x}). \tag{2}$$

In other words, considering all the feasible solutions, $F(\boldsymbol{w})$ represents the minimum value of maximal travel time. Furthermore, $F(\boldsymbol{w})$ is determined by the number of nodes n, the number of hubs p and link travel times $\boldsymbol{w}$. It is easy to verify that $F(\boldsymbol{w})$ is a strictly increasing function with regard to $w_{ij},\ i,j = 1,\ldots,n$. More precisely, for given $\boldsymbol{w} = (w_{ij})$ and $\boldsymbol{w}' = (w'_{ij}), F(\boldsymbol{w})$ has the following monotonicity,

(1) $F(\boldsymbol{w}) \leqslant F(\boldsymbol{w}')$ when $w_{ij} \leqslant w'_{ij}$ for all $i,j = 1,\ldots,n$;
(2) $F(\boldsymbol{w}) < F(\boldsymbol{w}')$ when $w_{ij} < w'_{ij}$ for all $i,j = 1,\ldots,n$.

Generally speaking, if some links' travel times increase, then the maximal travel time will not necessarily increase. However, if all the travel times increase, then it is certain that the maximal travel time will increase. As a result, the monotonicity of $F(\boldsymbol{w})$ is a natural thing.

If the travel times are not deterministic values, i.e., $\boldsymbol{w} = (w_{ij})$ being replaced with uncertain travel times $\boldsymbol{\eta} = (\eta_{ij}),\ i,j = 1,2,\ldots,n$, then model (1) will not work. This is because the travel time of path $i \to k \to m \to j$, namely, $(\eta_{ik} + \alpha\eta_{km} + \eta_{mj})$, is an uncertain variable, and we can not compare uncertain variables in a similar way to compare deterministic ones. Moreover, we may not find a p-hub location $\boldsymbol{x}^*$ which has a minimum value of the maximal travel time in all situations. In order to modify model (1) in uncertain environment, next we first consider the uncertainty distribution of the maximal travel time.

Since in the uncertain environment η_{ij} may take many different values, the minimum value of maximal travel time $F(\boldsymbol{\eta})$ may naturally take different values. As a result, a more meaningful way is to investigate the uncertainty distribution of $F(\boldsymbol{\eta})$, which is denoted by $\Psi(t)$, i.e.

$$\Psi(t) = \mathcal{M}\{F(\boldsymbol{\eta}) \leqslant t\},$$

where t is a positive real number.