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Finite element solution for a coal-bed methane reservoir model

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Abstract

We present a mathematical model for fluid flow in porous media in the case of coal bed methane reservoirs. The model consists of a two-phase immiscible fluid flow with gas and water and takes into account desorption. We discretize the problem by a finite element method with adding a streamline diffusion term for the case of small capillary pressure. Time integration uses the fully implicit Euler scheme. Numerical experiments in radial and fully 2-D cases show the effectiveness of the model and numerical method.

Keywords: Porous media; Two-phase flow; Finite elements

1. Introduction

In Reservoir Engineering, CBM (Coal Bed Methane) refers to the natural gas contained within coal. In the coal matrix gas is stored by adsorption, *i.e.* gas molecules adhere to the surface. The pressure/volume relationship is described by the Langmuir isotherm. When the initial gas content of the coal is below the equilibrium curve in this isotherm, no free gas is available and the fractures in the coal are filled with water. When pressure is lowered, a dewatering process starts before gas extraction. This suggests that mathematical modelling of such a process invokes a two-phase flow in a porous medium. More details on the CBM process and its modelling can be found in [2,7,18].

The present work aims at deriving a mathematical model that predicts the gas extraction process from a CBM reservoir. For this we shall consider classical fluid flow in porous medium of an immiscible mixture of gas and water, driven by the desorption process. Study of two-phase flows is widely used in reservoir engineering and applied mathematics literature (See [1,5,12,16] for instance, and the references therein). Our contribution here is the adjunction of desorption as a source for the gas mass conservation equation. There is also a specific issue to the presented modelling: Since in most applications we start by a dewatering process, we have to deal with a two-phase model that starts with only one phase. The resulting singularity at initial time imposes the use of a fully implicit time integration scheme. This results in a nonlinear system that is heavy and time-consuming to solve. Numerical experiments show indeed that any simpler technique (*e.g.* IMPES, Implicit Pressure Explicit Saturation) fails to converge.

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Modelling immiscible two-phase flows generally uses the saturation of one phase and a global pressure as variables (see for this [1,5,16] for instance). Some variants in the definition of a global pressure may appear in such modelling. We have adopted here the approach of [5], this one being valid for most configurations (for instance for either compressible, slightly compressible or incompressible flows).

For numerical approximation purposes, we resort in this work to a classical finite element method for space discretization. Although finite volume methods seem to be the most popular numerical methods for the solution of problems in reservoir engineering, some works like in [5,14,17] have used with success finite element methods (either primal or mixed ones). On the other hand, dedicated finite element methods have been designed for compressible and convection dominated flows (see *e.g.* [11]). We show in the present paper that the use of such methods with adequate treatment of convection terms gives accurate and efficient results. Moreover, the ability to use mesh adaption techniques allows dealing with heterogeneous media where high jumps in the permeability and in capillary pressure occur. We shall however mention that the effectiveness of the presented scheme for handling highly heterogeneous rocks has not yet been tested. We shall refer for this to more dedicated works such as [9,8,10,13] and the references therein.

Let us mention at this point that the radial 1-D model was eventually implemented as the forecasting engine within the commercial software DOT.CBM. Owing to its numerical performance it was also coupled successfully with a stochastic search engine to solve complex multi-well history-matching problems (see [15]).

The paper is organized as follows: We start by deriving the set of partial differential equations the govern the process, and give some aspects to consider when numerical solution is addressed. In Section 3, we consider the approximation of the obtained model by a finite element method. In particular, we show the necessity of using some upwinding treatment to deal with the case of low capillary pressure in our application and then the saturation is solution of a transport equation. After deriving the finite element differential system of equations, we consider its discretization in time by the backward Euler scheme. For this end, a simple adaptive time procedure is used. This is motivated by the singularity in initial time and also for efficiency reasons if a time dependent well pressure is used with large variations. Finally, Section 4 presents some numerical tests, first on a radial case, *i.e.* when we consider a unique well in a large reservoir and use radial symmetry and then for the fully two-dimensional case with multiple wells.

2. Model equations

To model fluid flow in a CBM reservoir, we consider an immiscible mixture of gas and water, where the gas is considered as compressible and the water as slightly compressible.

2.1. Derivation of the equations

We denote in the sequel by S_g and S_w the respective saturations of gas and water so that $S_g + S_w = 1$. In general the indices g and w will refer to quantities relative to gas and water respectively. Denoting by ϱ_a the density of each phase $a = g, w$ and by $\boldsymbol{v}_a$ its velocity, we have the mass conservation equations:

$$\frac{\partial}{\partial t}(\phi \varrho_w S_w) + \nabla \cdot (\varrho_w \boldsymbol{v}_w) = 0, \tag{2.1}$$

$$\frac{\partial}{\partial t}(\phi \varrho_g S_g) + \nabla \cdot (\varrho_g \boldsymbol{v}_g) = f_D, \tag{2.2}$$

where f_D is the rate of desorbed gas. For simplicity we denote by S the water saturation S_w and then $S_g = 1 - S$. We assume that in a porous medium the velocities obey to the Darcy law:

$$\boldsymbol{v}_a = -\frac{k_a}{\mu_a} \boldsymbol{K} \nabla p_a \quad a = g, w, \tag{2.3}$$

where k_a, μ_a and p_a are respectively the relative permeability, the viscosity and the pressure of each phase a and $\boldsymbol{K}$ is the tensor of absolute permeability. The relative permeabilities k_a are assumed as functions of the saturations and the gas and water viscosities μ_g and μ_w are considered constant. The capillary pressure is defined as a function of S by

$$p_c(S) = p_g - p_w,$$

where we assume that the function $p_c(S)$ is nonnegative and nonincreasing.