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Efficient absorbants in generalized de Bruijn digraphs☆

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A B S T R A C T

Let $D = (V, A)$ be a digraph with vertex set V and arc set A. An efficient absorbant (respectively, dominating set) of a digraph $D = (V, A)$ is a set $S \subseteq V$ such that, for every $v \in V \setminus S$, there exists exactly one out-neighbor (respectively, in-neighbor) of v in S and there is no arc in the induced digraph of S, where an out-neighbor (respectively, in-neighbor) of v in S is a vertex u with an arc (v, u) (respectively, (u, v)) in A with $u \in S$. The efficient absorbant conjecture is as follows: there is an efficient absorbant in generalized de Bruijn digraph $G_B(n, d)$ if and only if $\frac{n}{d+1}$ is a multiple of $\gcd(n, d-1)$, where $\gcd(x, y)$ denotes the greatest common divisor of integers x and y. The sufficient condition of this conjecture was proved in DOI:10.1080/00207160.2016.1154949. In this paper, we settle the conjecture. Moreover, we also show that, the subclasses in $G_B(n, d)$ having efficient absorbants and efficient dominating sets are equivalent.

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1. Introduction

Let $D = (V, A)$ denote a digraph in which V and A are the sets of vertices and arcs, respectively. For vertices $u, v \in V$, vertex u is said to be an *in-neighbor* (respectively, *out-neighbor*) of v if arc $(u, v) \in A$ (respectively, $(v, u) \in A$). An arc of the form (u, u) for some $u \in V$ is a *loop*. The *in-neighborhood* (respectively, *out-neighborhood*) of a vertex $v \in V$, denoted by $I(v)$ (respectively, $O(v)$), is the vertex set $I(v) = \{u : (u, v) \in A\}$ (respectively, $O(v) = \{u : (v, u) \in A\}$).

Definition 1.1. An *absorbant* (respectively, a dominating set) of a digraph $D = (V, A)$ is a set $S \subseteq V$ such that, for every $v \in V \setminus S$, there exists an arc $(v, u) \in A$ (respectively, $(u, v) \in A$) with $u \in S$. An *efficient*

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absorbant (respectively, efficient dominating set) S in D is an absorbant (respectively, a dominating set) S such that, for every $v \in V \setminus S$, there exists exactly one out-neighbor (respectively, in-neighbor) of v in S and there is no arc in the induced digraph of S.

Note that, in undirected graphs, an efficient dominating set is also called a *perfect code* [1,2]. The efficient domination problem has been extensively studied [3–12,2,13–15]. The absorbant problem which was introduced in [16] is a variant of the domination problem. As mentioned in [16], the absorbant problem can be applied to the file servers of a network system by regarding an absorbant as a set of file-servers so that every node in the network can access file-servers directly.

Generalized de Bruijn digraphs were introduced by Imase and Itoh [17,18] and, independently, also by Reddy, Pradhan, and Kuhl [19]. The generalized de Bruijn digraph $G_B(n,d)$ has vertex set $V(G_B(n,d)) = \{0,\dots,n-1\}$ and arc set

$$A(G_B(n,d)) = \{(x,y) : y \equiv dx + i \pmod n, 0 \leqslant i < d\}.$$

Note that if $n = d^m$, then $G_B(n,d)$ is the well-known de Bruijn digraph $B(d,m)$ [20]. Generalized de Bruijn digraphs have good logical network topologies and have been extensively studied [21–24,17,25,19,16,26–28].

Example 1. Fig. 1(a) depicts $G_B(8,3)$. In Fig. 1(b), we can find that the set $S_1 = \{1,6\}$ is an efficient absorbant of $G_B(8,3)$. Note that $I(1) = \{0,3,5\}$ and $I(6) = \{2,4,7\}$. Thus each vertex which is not in S_1 has exactly one arc incident to S_1. In Fig. 1(c), the set $S_2 = \{1,2\}$ is an efficient dominating set. We can find that each vertex not in S_2 is dominated exactly once by S_2.

In [16], Shan et al. posed an open question as follows: Find the sufficient conditions for the absorbant number of generalized de Bruijn digraphs $G_B(n,d)$ to be the lower bound $\lceil \frac{n}{d+1} \rceil$. In [27], Wang et al. conjectured that, for $G_B(n,d)$, there is an efficient absorbant if and only if $\frac{n}{d+1}$ is a multiple of $\gcd(n,d-1)$, where $\gcd(x,y)$ denotes the greatest common divisor of integers x and y. The sufficient condition of the efficient absorbant conjecture is affirmed in [28]. In this paper, we settle the efficient absorbant conjecture. Moreover, we also show that, the subclasses in $G_B(n,d)$ having efficient absorbants and efficient dominating sets are equivalent.

The rest part of this paper is organized as follows. In Section 2, we introduce some preliminaries of the efficient absorbant problem and efficient domination problem in generalized de Bruijn digraphs. Section 3 contains our proof for the necessary condition of the efficient absorbant conjecture. In Section 4, we show that there is an efficient dominating set in $G_B(n,d)$ if and only if $\frac{n}{d+1}$ is a multiple of $\gcd(n,d-1)$. We conclude in Section 5.

2. Preliminaries

In this section, we introduce some previous results and investigate some properties of efficient absorbants and dominating sets in $G_B(n,d)$. Hereafter, for integers x and y, we use $x|y$ to denote that x divides y.

Lemma 2.1 (*[27]*)**.** *If there is an efficient absorbant (respectively, efficient dominating set) in $G_B(n,d)$, then $d > 1$ and $(d+1)|n$.*

Proof. We only show that the lemma holds for efficient absorbants. By using a similar argument, we can also show that the lemma holds for efficient dominating sets. It is clear that there is no efficient absorbant when $d = 1$. Thus if there is an efficient absorbant in $G_B(n,d)$, then $d > 1$. Assume that S is an efficient absorbant in $G_B(n,d)$. By Definition 1.1, we have that $|S| + d|S| = |V|$. That is, $(d+1)|S| = n$. This further implies $(d+1)|n$. This completes the proof. $\square$

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