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Subsidizing purchases of public interest products: A duopoly analysis under a subsidy scheme

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ABSTRACT

We investigate a symmetric duopoly setting in which two manufacturers produce the traditional and public interest (PI) products under a government's subsidy scheme. A higher subsidy can increase the sale of the PI product but reduce the sale of the traditional product. Then, we study an asymmetric setting in which a manufacturer produces one of the two products and the other manufacturer produces both products. The government's optimal subsidy is increasing in the marginal externality of the PI product.

1. Introduction

In today's markets, there exist a variety of public interest (PI) products that possess functional attributes similar to traditional products. The PI products have a substantial social impact, as the consumption of these products can result in some direct outcomes as expected, and also generate indirect, or involuntary, benefits to the persons or firms who do not use the products. For example, an energy efficient ("green") air conditioner is a typical PI product, because the air conditioner can directly serve those persons who use it, and also benefits the society by consuming less energy than traditional air conditioners. An electric vehicle is another PI product, because it can help reduce carbon emissions compared with traditional vehicles. The electric vehicle benefits not only its driver but also all the persons around the vehicle. As Ovchinnikov and Raz [7] mentioned, other PI products include the energy efficient appliances (e.g., water-saving toilets and energy-saving lamp), the eco-consumables (e.g., organic fertilizer), etc.

Observing the substantial social value of the PI products, some governments have implemented incentive schemes to stimulate purchases of the PI products. The most often used one is the consumer subsidy scheme under which a government awards a subsidy to each consumer who buys a PI product. The subsidy scheme mainly aims at improving the affordability of PI products and maximizing the social welfare. Some governments have subsidized consumers, encouraging them to buy electric vehicles. For instance, Desk [2] reported that in September 2011, the Swedish government approved a subsidy program to provide a subsidy of 40,000 kr per car for purchases of electric cars and other "super green cars" with ultra-low carbon emissions since January 2012. An *interesting* research question arises as follows: Can a subsidy scheme help reduce the selling price and increase the sales of the PI products? It thus behooves us to investigate manufacturers' pricing decisions and profits as well as the sales of the PI products under a government's subsidy scheme.

In practice, a manufacturer may produce only the traditional product, only the PI product, or both products. For example, some automobile manufacturers (e.g., Mazda and Chrysler) only make the fuel vehicles (traditional products), some manufacturers (e.g., Fisker Automotive and Tesla Motors) only focus on the production of electric vehicles (PI products), and the others (e.g., Toyota and Honda) produce both fuel vehicles and electric vehicles. First, in Section 2, we consider a symmetric duopoly setting in which two manufacturers make both the traditional and the PI products. For the game, we obtain the two manufacturers' pricing decisions in Nash equilibrium, construct a social welfare function for the government, and maximize the social welfare to find the optimal subsidy. For the symmetric

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duopoly setting, we also analyze the case in which each manufacturer produces only the PI product. Then, in Section 3, we investigate an asymmetric duopoly setting in which a manufacturer produces either the traditional product or the PI product, whereas the other manufacturer produces both the traditional and the PI products. For the setting, we also obtain the pricing decisions in Nash equilibrium, and maximize the social welfare to find the optimal subsidy for the government.

Some relevant publications are concerned with a government's subsidy scheme for the adoption of green technologies. For example, Huang et al. [5] investigated an automobile scrappage program under which a government awards a subsidy to each consumer who trades in his or her used automobile with a new fuel-efficient automobile. Chan, Leng, and Liang [1] examined the impact of a government's tax reduction policies on the sales of new automobiles and the profits of manufacturers. Luo et al. [6] investigated a government's price-discount incentive scheme that involves a price discount rate and a subsidy ceiling. Those publications mainly focus on the impact of a government's subsidy without considering the competition between traditional and energy-efficient products. Moreover, the relevant publications did not consider the externality of the PI products, which captures the most important feature of such products. Different from them, we incorporate the externality into the social welfare function. A future research direction is suggested in Section 4, and the proofs of propositions are relegated to online Appendix A.

2. Game-theoretic analysis and social welfare in a symmetric duopoly setting

We consider a symmetric duopoly setting in which two manufacturers (i.e., manufacturer i, $i = 1, 2$) make and sell the same or similar products to consumers in a market. That is, in the setting, both manufacturers produce the traditional and the PI products. The two manufacturers hold different brands for their products, which possess similar quality, function, and other attributes. A government implements a subsidy scheme to promote the PI products made by the two manufacturers. Since this paper is concerned with the PI products, we focus on the two-product case in which each manufacturer makes the two types of products and the PI-product case in which only the PI products are produced.

For each case, we analyze a "simultaneous-move" game in which the two manufacturers make their pricing decisions under the subsidy scheme. Then, we compute the social welfare that is generated by implementation of the subsidy scheme and maximize it for the government's optimal subsidy.

2.1. The duopoly analysis with two products

We consider a symmetric duopoly setting in which the two manufacturers sell their traditional and PI products to compete for consumers.

2.1.1. The sales functions

Under a subsidy scheme, the government provides a consumer with a subsidy \$$s$ when the consumer purchases a PI product. If the consumer decides to buy a traditional product from manufacturer i ($i = 1, 2$), then he or she needs to pay \$$p_i$; otherwise, the consumer should make his or her own payment \$$(\hat{p}_i - s)$ to buy a PI product from manufacturer i. Hereafter, to simply distinguish the two product types, we use "ˆ" to indicate the PI product.

We consider the following inverse sales curve for each product made by manufacturer i:

$$\begin{cases} p_i = \theta - (q_i + \beta q_{3-i}) - \gamma(\hat{q}_i + \beta \hat{q}_{3-i}), \\ \hat{p}_i - s = \theta - (\hat{q}_i + \beta \hat{q}_{3-i}) - \tilde{\gamma}(q_i + \beta q_{3-i}), \end{cases} \tag{1}$$

where θ denotes the possible highest price; p_i ($\hat{p}_i$) and q_i ($\hat{q}_i$) are the price and the sale for manufacturer i's traditional (PI) product, respectively; β reflects the substitutability (competition) between the two manufacturers; and γ ($\tilde{\gamma}$) represents the degree of substituting the PI (traditional) product for the traditional (PI) product. As the value of γ ($\tilde{\gamma}$) increases, a consumer is more willing to accept the PI (traditional) product. Similar inverse sales/demand functions have been widely used by many scholars in the economics and operations management areas to analyze various competition-related issues with two or more products; see, e.g., Dobson and Waterson [3,4]. Solving equations in (1) for q_i and $\hat{q}_i$ ($i = 1, 2$) yields the sales functions as

$$\begin{cases} q_i = \dfrac{(1-\gamma)(1-\beta)\theta - p_i + \gamma(\hat{p}_i - s) + \beta[p_{3-i} - \gamma(\hat{p}_{3-i} - s)]}{(1-\beta^2)(1-\gamma\tilde{\gamma})}, \\ \hat{q}_i = \dfrac{(1-\tilde{\gamma})(1-\beta)\theta - (\hat{p}_i - s) + \tilde{\gamma} p_i + \beta[(\hat{p}_{3-i} - s) - \tilde{\gamma} p_{3-i}]}{(1-\beta^2)(1-\gamma\tilde{\gamma})}. \end{cases} \tag{2}$$

2.1.2. Game-theoretic analysis under a subsidy scheme

Since the two manufacturers' products possess similar attributes, they incur an identical unit acquisition cost for each product. Accordingly, we denote each manufacturer's unit cost of the traditional product and that of the PI product by c and $\hat{c}$, respectively. Manufacturer i's profit drawn from selling a traditional (PI) product is $p_i - c$ ($\hat{p}_i - \hat{c}$). The manufacturer's total profit generated from the sales of both products can be calculated as $\pi_i = (p_i - c)q_i + (\hat{p}_i - \hat{c})\hat{q}_i$, which, using (2), can be specified as

$$\begin{aligned} \pi_i = \frac{1}{J}\{&(p_i - c)[(1-\gamma)(1-\beta)\theta - p_i + \gamma(\hat{p}_i - s) + \beta p_{3-i} - \beta\gamma(\hat{p}_{3-i} - s)] \\ &+ (\hat{p}_i - \hat{c})[(1-\tilde{\gamma})(1-\beta)\theta - (\hat{p}_i - s) + \tilde{\gamma} p_i + \beta(\hat{p}_{3-i} - s) - \beta\tilde{\gamma} p_{3-i}]\}, \end{aligned} \tag{3}$$

where $J \equiv (1-\beta^2)(1-\gamma\tilde{\gamma})$.