Operations Research Letters 45 (2017) 415–420

Contents lists available at ScienceDirect

Operations Research Letters

journal homepage: www.elsevier.com/locate/orl

Optimal pricing for selling to a static multi-period newsvendor

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ARTICLE INFO

Article history:
Received 2 December 2016
Received in revised form 16 March 2017
Accepted 26 May 2017
Available online 17 June 2017

Keywords:
Newsvendor
Pricing
Supply chain management

ABSTRACT

This paper considers a multi-period supply chain model in which a supplier sells to a multi-period newsvendor. Such a problem is relevant in industries with long production lead times. We study the optimal pricing problem for the supplier. We derive procedures for solving the optimal prices and show that the optimal pricing sequence is decreasing in time. We also show that the optimal prices are increasing in the backorder cost when the cumulative demand functions have increasing generalized failure rates.

1. Introduction

The newsvendor model is probably the most classical model in inventory and supply chain management. In a newsvendor model, a retailer decides the order quantity of a product when facing uncertain demand, trading off the possibility of overstocking with understocking. There have been extensive studies on newsvendor models in the literature. For some recent reviews, we refer the readers to [7] and [14].

In practice, the newsvendor (retailer) is often supplied by a supplier who has pricing power over the product. The traditional newsvendor model assumes that the purchase price of the product is fixed. However, the supplier may utilize its pricing power to decide a price that maximizes his profit. Such a model is first studied in [10], in which the authors consider a supplier who sells to a newsvendor and derives the optimal price for the supplier. In this paper, we extend the model in [10] to a multi-period setting. We consider a supplier who sells a product to a multi-period newsvendor. Inventory holding cost and backorder cost are incurred if there are excess or a shortage of inventory at the end of each period. The supplier posts a price for each period in the beginning of the time horizon, and the retailer must commit to an ordering quantity for each period before the selling horizon starts.

There are many practical situations where such a multi-period model is relevant. One example is in the fresh produce industry. One major characteristic of the fresh produce industry is the long lead time of the production due to the growing cycle of the produce. Therefore, the retailer's order quantities must be planned ahead of the growing season despite demand uncertainty so that the supplier can deliver the desired quantity during the selling season. Moreover, due to the short shelf lives of fresh produce and the high holding costs, it is not possible to have all products delivered at once and stored during the selling season. Indeed, it is common practice in the fresh produce industry for the retailers to commit to a sequence of ordering quantities before the selling season starts. According to i2, a supply chain results company, Dole Asia (a division of Dole Food Company) plans the plant according to the customer's demand [3]; and Tesco, the UK's largest retailer, signs three- to five-year contract with its fresh produce suppliers, allowing them to plan further ahead [8]. In addition, the supplier sometimes has the pricing power over the product due to the scarcity of supply (e.g., see [15]). In such situations, the problem for the supplier is one that sells to a multi-period newsvendor as described in our model.

In this paper, we solve the problem of optimal pricing for selling to a multi-period newsvendor. Particularly, we provide a simple solution procedure for the optimal prices as well as a closed-form solution to the optimal ordering quantities under the optimal prices. We find that the optimal prices are decreasing in time under our model. We also adopt the analysis in [10] to obtain comparative statics for the optimal prices.

In the remainder of this section, we review related literature. First, our model is related to the inventory theory. Various inventory models have been studied and many results have been obtained, see, e.g., [12,13] and [17]. However, instead of the typical continuous review or periodic review models, we consider a case where all the inventory decisions must be made at the beginning (which is equivalent to having a lead time that is longer than the planning horizon). Therefore, our decision model invokes only one

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http://dx.doi.org/10.1016/j.orl.2017.05.006
0167-6377/

shot of inventory decisions for the retailer rather than decisions that depend on the state of the system.

Our work is also related to the revenue management literature (see [18] for a comprehensive review). In revenue management, the seller decides the optimal prices for its products to maximize the revenue, which is the case for the supplier in our model. However, in our model, the demand function (i.e., the purchase quantity of the retailer) is resulting from solving a multi-period newsvendor model. Therefore, our problem is a very specific form of pricing problem.

There has also been a wealth of literature of supply chain contracts in which the supplier and retailer sign a contract in terms of cost-sharing, buy-back, etc, to coordinate the supply chain and to increase the overall efficiency [4,9,19]. Although coordinated supply chain performs better in many aspects, in practice, a non-coordinated supply chain is still widespread due to its simple structure. Our paper will be focused on non-coordinated supply chains.

Finally, as we mentioned in the introduction, one application of this model is in the fresh produce industry. There has been much work that is related to the fresh produce supply chain. We refer the readers to [1] for a thorough review. However, in most of those studies, the supply chain is modeled by a dynamic system in which the lead time is relatively short compared to the selling season. For example, [5,6,21] propose approximation algorithms for perishable inventory systems under different settings. However, those assumptions may not be realistic in practice because of the long growing cycle of the produce and the limited spot market [11]. In contrast, we consider the case where the supplier and the retailer only make a one-time decision about the price and the order quantity ahead of the selling season.

2. Model

We consider a supplier who provides the supply of a certain product to a retailer over a horizon of T periods. Before the start of the time horizon, the supplier decides the selling price in all time periods p_t, for $t = 1, 2, \dots, T$. After observing the supplier's prices, the retailer decides the order quantities Q_t for each period. In each period t, the retailer faces a random demand D_t with $F_t(\cdot)$ being the cumulative distribution function. We do not assume that the demands in each period are independent. The retailer incurs a per unit holding cost h to carry inventory from one period to the next, and a per unit backorder cost b for each unit of unsatisfied demand. In addition, the selling price of the retailer is r. In our model, we assume the holding cost, the backorder cost, and the selling price are constant throughout the entire horizon. However, our model can be easily extended to the case where they differ across different periods.

In our model, before the start of the first period, the supplier decides a sequence of prices $\mathbf{p} = \{p_t\}_{t=1}^T$. Then the retailer decides $\mathbf{Q} = \{Q_t\}_{t=1}^T$ based on $\mathbf{p}$. The initial inventory of the retailer is $I_0 = 0$. Then, in each period t, the following sequence of events take place.

1. The retailer receives order quantity Q_t and pays the supplier $p_t Q_t$.
2. Demand D_t realizes, and the inventory at the end of period t is $I_t = I_{t-1} + Q_t - D_t$.
3. If $t \le T-1$, then the profit in period t is $R_t = rD_t - hI_t^+ - bI_t^-$, where $I_t^+ = \max\{I_t, 0\}$ represents the positive part of I_t, and $I_t^- = \max\{-I_t, 0\}$ represents the negative part of I_t. In the last period T, the profit is $R_T = r\min\{D_T, I_{T-1}+Q_T\} - hI_T^+ - bI_T^-$.

Here, we can view the backorder cost as the per period loss of goodwill cost when the seller is unable to satisfy the demand (regardless of whether the demand can be satisfied eventually). The total profit of the retailer is $R(\mathbf{Q}) = \sum_{t=1}^T (R_t - p_t Q_t)$, and the total revenue for the supplier is $\sum_{t=1}^T p_t Q_t$.

2.1. The retailer's problem

Having observed the supplier's prices p_t, the retailer decides how many units to purchase in each period before demand arrives. The retailer's problem can be modeled as follows:

$$
\begin{aligned}
\max \quad & R(\mathbf{Q}) = \mathbb{E}\left[r \min\left\{ \sum_{t=1}^T Q_t, \sum_{t=1}^T D_t \right\}\right] \\
& \quad - \sum_{t=1}^T \mathbb{E}\left[hI_t^+ + bI_t^- + p_t Q_t \right] \qquad (1)\\
\text{s.t.} \quad & I_0 = 0,\ I_t = I_{t-1} + Q_t - D_t, \qquad \forall t = 1, \dots, T \\
& Q_t \ge 0, \qquad \forall t = 1, \dots, T.
\end{aligned}
$$

Here, the first part of the objective function is the expected revenue, and the second part of the objective function is the expected cost. We can rewrite the objective function as follows:

$$
r\mathbb{E}\left[\sum_{t=1}^T D_t\right] - r\mathbb{E}\left[\sum_{t=1}^T D_t - \sum_{t=1}^T Q_t\right]^+ - \sum_{t=1}^T \mathbb{E}\left[hI_t^+ + bI_t^- + p_t Q_t \right].
$$

Note that the first term is a constant. Therefore, we can write (1) as a cost minimization problem.

$$
\begin{aligned}
\min \quad & r\mathbb{E}\left[I_T^-\right] + \sum_{t=1}^T \mathbb{E}\left[hI_t^+ + bI_t^- + p_t Q_t \right] \\
\text{s.t.} \quad & I_0 = 0,\ I_t = I_{t-1} + Q_t - D_t, \qquad \forall t = 1, \dots, T \\
& Q_t \ge 0, \qquad \forall t = 1, \dots, T.
\end{aligned}
$$

Then, using the inventory dynamics $I_t = \sum_{i=1}^t (Q_i - D_i)$ to replace the I_t's in the objective function, we can rewrite the cost minimization problem as

$$
\begin{aligned}
\min \quad & C(\mathbf{Q}) = r\mathbb{E}\left[\sum_{i=1}^T (Q_i - D_i)\right]^- \\
& + \sum_{t=1}^T \mathbb{E}\left[h\left(\sum_{i=1}^t (Q_i - D_i)\right)^+ + b\left(\sum_{i=1}^t (Q_i - D_i)\right)^- + p_t Q_t \right] \qquad (2)\\
\text{s.t.} \quad & Q_t \ge 0, \quad \forall t = 1, \dots, T.
\end{aligned}
$$

In this paper, we assume that D_i follows a continuous distribution with support on $[0, +\infty)$. Therefore, $C(\mathbf{Q})$ is a strictly convex function in $(Q_1, \dots, Q_T)$ on $Q_i \ge 0$. Thus, for any given $\mathbf{p}$, there is a unique optimal solution $\mathbf{Q}(\mathbf{p})$ for the retailer. In the following, we use $\mathbf{Q}^*(\mathbf{p})$ to denote the optimal purchasing strategy of the retailer under price $\mathbf{p}$.

2.2. The supplier's problem

In our model, we assume the supplier knows the retailer's ordering strategy and cost parameters. The supplier's problem is to find the optimal pricing policy to maximize the revenue under the retailer's ordering strategy. The supplier's problem can be written as follows:

$$
\begin{aligned}
\max \quad & S(\mathbf{p}) = \sum_{t=1}^T p_t Q_t^*(\mathbf{p}) \qquad (3)\\
\text{s.t.} \quad & p_t \ge 0, \quad \forall t = 1, \dots, T.
\end{aligned}
$$