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Equivalent conditions for the existence of an efficient equilibrium in coalitional bargaining with externalities and renegotiations

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ABSTRACT

We consider a noncooperative coalitional bargaining game with externalities and renegotiations. We provide the necessary and sufficient condition for an efficient stationary subgame perfect equilibrium to exist. This condition states that a Nash bargaining solution is immune to any blocking.

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1. Introduction

Gomes [3] provided a sufficient condition for an efficient stationary subgame perfect equilibrium (SSPE) to exist in coalitional bargaining with externalities and renegotiations. Gomes's [3] model has four features: (i) players repeat negotiations about forming coalitions; (ii) players obtain an instantaneous payoff in every bargaining round; (iii) externalities occur among coalitions (the bargaining situation is described as a partition function game); and (iv) a proposer is randomly selected for each round. An efficient SSPE is one where a grand coalition forms immediately. Okada [10] derived a necessary and sufficient condition without externalities. However, a necessary and sufficient condition with externalities and renegotiations has not been shown.

Given an arbitrary discount factor, we provide a necessary and sufficient condition for an efficient SSPE to exist in coalitional bargaining with externalities and renegotiations. From the necessary and sufficient condition, an efficient SSPE can be fully characterized for any discount factor. Gomes [3] provided a sufficient condition for an efficient SSPE to exist for any discount factor. Even if Gomes's sufficient condition is not satisfied, an efficient SSPE always exists when a discount factor is small enough. We also show that Gomes's condition is necessary and sufficient for an efficient SSPE to exist for any discount factor.

Moreover, we give a cooperative-game-theoretic interpretation to the condition for an efficient SSPE to exist for any discount factor. The condition states that a Nash bargaining solution (NBS) is "bargaining-blocking-proof" in the following sense. Under a coalition structure (partition of the set of players), each coalition bargains over the worth of the grand coalition, its disagreement payoff being its worth, and an NBS under the coalition structure is given by the payoff tuple that maximizes the weighted product of net payoffs for coalitions over their disagreement payoffs. Here, by integrating and forming a new coalition, some coalitions can induce a new coalition structure as well as a new NBS under this coalition structure. If the sum of initial NBS payoffs for these coalitions is less than the new NBS payoff for the integrated coalition, these coalitions block the initial NBS. The NBS (tuple of NBSs under coalition structures) is said to be *bargaining-blocking-proof* if an NBS is not blocked by any coalition under any coalition structure.

We refer to other related literature on noncooperative coalitional bargaining, where renegotiations are allowed except for Kawamori and Miyakawa [8]. Gomes [4] analyzed the same noncooperative bargaining game as ours for a three-player case. He found four patterns of dynamic processes to the grand coalition and characterized the SSPE payoffs in the limit as a discount factor tends to unity. Seidmann and Winter [12] were the first to present a noncooperative coalitional bargaining game with renegotiations (they called it the "reversible actions" model), which is a rejector-propose model. They provided some examples of gradual coalition formation as well as immediate move toward grand coalition in a model without externalities. Gomes and Jehiel [5] considered a

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general case where coalitions may break up and have externalities. They provided a necessary and sufficient condition for convergence to the efficient state. Bloch and Gomes [2] considered a repeated coalitional bargaining game where the coalitions endogenously choose whether or not to exit under externalities among coalitions. Hyndman and Ray [6] considered nonstationary subgame perfect equilibria for a bargaining game. Kawamori and Miyakawa [8] provided a necessary and sufficient condition for an efficient SSPE to exist with externalities and rejecter-exist partial breakdowns but without renegotiations. Owing to the partial breakdowns, the condition of Kawamori and Miyakawa [8] is quite different from the condition of the present paper. We leave the characterization of inefficient SSPEs where the grand coalition does not form immediately to a future study. Gomes [4] attempted it in a three-player case.

The paper is organized as follows. Section 2 defines a noncooperative coalitional bargaining game. Section 3 provides necessary and sufficient conditions for an efficient SSPE to exist and gives a cooperative-game-theoretic interpretation of the conditions. Section 4 presents some applications. All proofs are relegated to the supplement (Kawamori and Miyakawa [9]).

2. The model

2.1. Partition function game

Let (N, v) be a partition function game; that is, a pair (N, v) such that N is a nonempty finite set and v is a function from $\mathcal{C} := \left\{(S, \pi) \in 2^N \times \Pi | S \in \pi\right\}$ to $\mathbb{R}_+$, where Π is a set of partitions of N. An element of N is called a *player*, a nonempty subset of N is called a *coalition*, and a partition of N is called a *coalition structure*. $v(S, \pi)$ represents the worth of coalition S under coalition structure π. For convenience, for any function f from $\mathcal{C}$ and any $(S, \pi) \in \mathcal{C}$, we write f_S^π instead of $f(S, \pi)$. We assume that the grand coalition is strictly efficient in (N, v); that is, for any $\pi \in \Pi$, $v_N^{\{N\}} > \sum_{S\in\pi} v_S^\pi$.

Let $p : \mathcal{C} \to \mathbb{R}_+$ such that for any $\pi \in \Pi$, $\sum_{I\in\pi} p_I^\pi = 1$. p represents the weights of the NBS as well as bargaining protocol in the extensive game defined below. For any $\pi \in \Pi$ and nonempty subset ρ of π, let $\pi|\rho := \pi \setminus \rho \cup \left\{\bigcup \rho\right\}$, where $\bigcup \rho = \bigcup_{S\in\rho} S$. $\pi|\rho$ is a coalition structure where coalition $\bigcup \rho$ forms under π.

2.2. Nash bargaining solution

We next define the Nash bargaining solution.

Definition 1. A *Nash bargaining solution (NBS)* is $b : \mathcal{C} \to \mathbb{R}$ such that for any $\pi \in \Pi$, b^π is a solution of $\max_{x\in\mathbb{R}^\pi} \prod_{I\in\pi} \left(x_I - v_I^\pi\right)^{p_I^\pi}$ s.t. $\forall I \in \pi \left(x_I^\pi \geq v_I^\pi\right) \wedge \sum_{I\in\pi} x_I \leq v_N^{\{N\}}$.

In an NBS, at any $\pi \in \Pi$, the Nash product $\left(x_I - v_I^\pi\right)^{p_I^\pi}$ is maximized under disagreement point $\left(v_I^\pi\right)_{I\in\pi}$ and feasibility $\sum_{I\in\pi} x_I \leq v_N^{\{N\}}$. Note that under the assumption of transferable utilities, there uniquely exists an NBS b, and for any $(I, \pi) \in \mathcal{C}$,

$$b_I^\pi = p_I^\pi \left(v_N^{\{N\}} - \sum_{J\in\pi} v_J^\pi\right) + v_I^\pi. \quad (1)$$

b_I^π is interpreted as the payoff for the owner of coalition I under coalition structure π.

We define a property of the NBS such that no coalition can block the NBS.

Definition 2. b is *bargaining-blocking-proof* if for any $\pi \in \Pi$ and nonempty $\rho \subset \pi$, $b_{\bigcup\rho}^{\pi|\rho} \leq \sum_{I\in\rho} b_I^\pi$.

When coalitions in ρ obtain a sufficiently large disagreement payoff $v_{\bigcup\rho}^{\rho|\pi}$ relative to coalitions in $\pi \setminus \rho$ by forming $\bigcup \rho$, they have such a large NBS payoff $b_{\bigcup\rho}^{\pi|\rho}$ that the inequality in Definition 2 does not hold; thus, they are better off by forming $\bigcup \rho$; hence, they block the NBS b^π. Bargaining-blocking-proofness means that such blocking is impossible. Mathematically, Definition 2 means that b is sub-additive.

Example 1 indicates that as externalities are more positive, the NBS is more likely to be bargaining-blocking-proof.

Example 1. Suppose that $N = \{1, 2, 3\}$: there are three players 1, 2, and 3. Let $\underline{\pi} := \{\{1\}, \{2\}, \{3\}\}$: $\underline{\pi}$ is the coalition structure with singleton coalitions. For any $I \in \underline{\pi}$, let $\pi_I := \{I, N \setminus I\}$: π_I is a coalition structure with a singleton coalition and a two-player coalition. Suppose that for any $I \in \underline{\pi}$, $v_N^{\{N\}} = 6$, $v_{N\setminus I}^{\pi_I} = 3$, $v_I^{\pi_I} = 1+e$ for some $e \in [-1, 1]$, and $v_I^{\underline{\pi}} = 1$: $e > 0$ $(e < 0)$, that is, $v_I^{\pi_I} > v_I^{\underline{\pi}}$ $(v_I^{\pi_I} < v_I^{\underline{\pi}})$ means positive (negative) externalities. Suppose that for any $(I, \pi) \in \mathcal{C}$, $p_I^\pi = \frac{1}{|\pi|}$: each coalition has an equal weight for the NBS.

For any $I \in \underline{\pi}$, $b_{N\setminus I}^{\pi_I} = \frac{1}{2}(6 - 3 - (1 + e)) + 3 = 4 - \frac{1}{2}e$, and $b_I^{\underline{\pi}} = \frac{1}{3}(6 - 3 \cdot 1) + 1 = 2$. Thus, b is bargaining-blocking-proof if and only if $\left(\forall I \in \underline{\pi}\right) b_{N\setminus I}^{\pi_I} \leq \sum_{J\in\underline{\pi}\setminus\{I\}} b_J^{\underline{\pi}} \iff 4 - \frac{1}{2}e \leq 2 + 2 \iff e \geq 0$. As externalities are more negative, the benefit from forming a two-player coalition is greater, and thus, the NBS is less likely to be bargaining-blocking-proof.

2.3. Extensive game

Let $\mathcal{S}$ be a set of $(\pi, A) \in \Pi \times 2^N$ such that A is a complete system of representatives for π, that is, there exists a bijection $f : A \to \pi$ such that for any $i \in A$, $i \in f(i)$. While an element in $\mathcal{S}$ is called a *state*, under state (π, A), an element in A is called an *active player*. For any $I, A \in 2^N$ such that $I \cap A$ is a singleton, let I_A be a unique element in $I \cap A$. I_A represents the active player that holds coalition I.

For any $\delta \in [0, 1)$, let $G(\delta)$ be an extensive game defined as follows. The underlying bargaining situation is represented by the partition function game (N, v). In a round at state (π, A), the bargaining proceeds as follows.

(i) Player I_A is selected as a proposer with a probability of p_I^π.
(ii) The player proposes a pair (ρ, t) such that $I \in \rho \subset \pi$, $t \in \mathbb{R}^\rho$, and $\sum_{J\in\rho} t_J = 0$. ρ is a set of I and some other coalitions. Now, proposing a subset ρ of the coalition structure π means proposing coalition $\bigcup \rho$. t is a transfer system for players J_A $(J \in \rho)$; that is, t_J $(J \neq I)$ is a monetary transfer from player I_A to player J_A and $-t_I = \sum_{J\in\rho\setminus\{I\}} t_J$ is the sum of such transfers.
(iii) Each player J_A $(J \in \rho)$ accepts or rejects the proposal in accordance with some predetermined order.
(iv) The state transits to $\left(\pi', A'\right)$ as follows.
  (a) If all players accept the proposal, for any $J \in \rho \setminus \{I\}$, player J_A gets t_J from player I_A, cedes his coalition J to player I_A, and leaves the game, while player I_A remains in the game as a representative of coalition $\bigcup \rho$. The state transits to $\left(\pi', A'\right) = \left(\pi|\rho, A \setminus \{J_A | J \in \rho\} \cup \{I_A\}\right)$.
  (b) If some player rejects the proposal, no transaction takes place. The state remains unchanged; that is, $\left(\pi', A'\right) = (\pi, A)$.

For any $J \in \pi'$, player $J_{A'}$ obtains his per period payoff $(1 - \delta) v\left(J, \pi'\right)$, where δ is the discount factor. Then, the game goes to the next round.

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