Journal of Statistical Planning and Inference 192 (2018) 1–17

Contents lists available at ScienceDirect

Journal of Statistical Planning and Inference

journal homepage: www.elsevier.com/locate/jspi

On estimation of limiting variance of partial sums of functions of associated random variables

Mansi Garg *, Isha Dewan

Indian Statistical Institute, New Delhi-110016, India

ARTICLE INFO

Article history:
Received 4 October 2016
Received in revised form 21 April 2017
Accepted 2 August 2017
Available online 12 August 2017

Keywords:
Associated random variables
Circular block bootstrap
Subsampling
U-statistics

ABSTRACT

We discuss three different estimators for estimating the limiting variance of partial sums of functions of associated random variables. The first two estimators are based on a Subsampling method, while the third is obtained using Circular block bootstrap. As an application, we also obtain estimators for the limiting variance for U-statistics based on stationary associated random variables.

© 2017 Elsevier B.V. All rights reserved.

1. Introduction

There exist several instances when the underlying random variables of interest are not independent. For example, the components of the moving average process $\{X_n, n \geq 1\}$ defined as $X_n = a_0\epsilon_n + \cdots + a_q\epsilon_{n-q}$, where $\{\epsilon_n\}_n$ are independent random variables, and $a_0, \ldots, a_q$ have the same sign are dependent; in reliability studies the lifetimes of components in structures in which components share the load so that failure of one component results in increased load on each of the remaining components are dependent. In both these examples the random variables are not independent but associated. Associated random variables are defined as follows.

Definition 1.1 (*Esary et al., 1967*). A finite collection of random variables $\{X_j, 1 \leq j \leq n\}$ is said to be associated, if for any choice of component-wise nondecreasing functions $k_1, k_2 : \mathbb{R}^n \to \mathbb{R}$, we have,

$$Cov(k_1(X_1, \ldots, X_n), k_2(X_1, \ldots, X_n)) \geq 0$$

whenever it exists. An infinite collection of random variables $\{X_j, j \geq 1\}$ is associated if every finite sub-collection is associated.

Apart from the reliability and survival studies, applications of associated random variables can also be found in statistical mechanics, percolation theory and interacting particle systems. Any set of independent random variables is associated, and nondecreasing functions of associated random variables are associated (cf. Esary et al., 1967). A detailed presentation of the asymptotic results and examples relating to associated sequences can be found in Bulinski and Shashkin (2007), Bulinski and Shashkin (2009), Oliveira (2012) and Prakasa Rao (2012).

* Corresponding author.
E-mail addresses: mansibirla@gmail.com (M. Garg), isha@isid.ac.in (I. Dewan).

http://dx.doi.org/10.1016/j.jspi.2017.08.002
0378-3758/© 2017 Elsevier B.V. All rights reserved.

Let $\mathbf{X} = \{X_n, n \geq 1\}$ be a sequence of associated random variables (not necessarily stationary). For each $j \geq 1$, let A_j be a finite subset of $\mathbb{N}$ with cardinality denoted as $\#A_j$, $g_j : \mathbb{R}^{\#A_j} \to \mathbb{R}$, and

$$Y_j = g_j(\mathbf{X}_{A_j}), \tag{1.1}$$

where $\mathbf{X}_{A_j} = \{X_i, i \in A_j\}$. We assume that for each $j \geq 1$, there exists a $\tilde{g}_j$ such that $g_j \ll \tilde{g}_j$, where $\tilde{g}_j : \mathbb{R}^{\#A_j} \to \mathbb{R}$. The relation "$\ll$" is defined as follows:

Definition 1.2 (*Newman, 1984*)**.** If g and $\tilde{g}$ are two real-valued functions on $\mathbb{R}^m$, $m \in \mathbb{N}$, then $g \ll \tilde{g}$ iff $\tilde{g} + g$ and $\tilde{g} - g$ are both coordinate-wise nondecreasing.

If $g \ll \tilde{g}$, then $\tilde{g}$ will be coordinate-wise nondecreasing.

Further, define

$$\tilde{Y}_j = \tilde{g}_j(\mathbf{X}_{A_j}), j \geq 1. \tag{1.2}$$

Let g_j, $\tilde{g}_j$, and A_j, for all $j \geq 1$ be such that $\{Y_j, j \geq 1\}$ and $\{\tilde{Y}_j, j \geq 1\}$ are stationary sequences. Under the condition $\sum_{j=1}^{\infty} |Cov(Y_1, Y_j)| < \infty$, the limiting variance of partial sums of $\{Y_j, j \geq 1\}$ is,

$$\sigma_g^2 = \lim_{n\to\infty} Var\left(\sum_{j=1}^{n} g_j(\mathbf{X}_{A_j})\right) \Big/ n = Var(Y_1) + 2\sum_{j=2}^{\infty} Cov(Y_1, Y_j). \tag{1.3}$$

Assume $\sigma_g^2 > 0$.

In this paper, we look at following three estimators of σ_g^2. Under suitable conditions on the moment and covariance structure of $\{Y_j, j \geq 1\}$ and $\{\tilde{Y}_j, j \geq 1\}$, these estimators are shown to be consistent.

- (PS) Peligard and Suresh (1995) obtained a consistent estimator for $\lim_{n\to\infty} (Var(\sum_{j=1}^{n} X_j)/n)$, where $\{X_n, n \geq 1\}$ is a sequence of stationary associated random variables. Their estimator is based on overlapping subseries of the underlying sample $\{X_i, 1 \leq i \leq n\}$. Using the same technique we obtain a consistent estimator of σ_g^2.
- (PR) The second estimator is based on the estimator of the limiting variance of mean discussed in Politis and Romano (1993). This estimator is also based on overlapping subseries of the underlying sample. They proved the consistency of the estimator when the underlying sample is from a mixing sequence. We extend their results to show consistency of the estimator when the sample is from $\{Y_j, j \geq 1\}$.
- (CBB) The third estimator is based on observations obtained using Circular Block Bootstrap (introduced in Politis and Romano, 1992).

We also obtain estimators for the limiting variance for U-statistics based on stationary associated random variables. Assume $\{X_n, n \geq 1\}$ is a sequence of stationary associated random variables and F is the common univariate distribution function. Define the U-statistic, $U_n(\rho)$, based on $\{X_j, 1 \leq j \leq n\}$, where $\rho : \mathbb{R}^2 \to \mathbb{R}$ is a symmetric function of degree two, by

$$U_n(\rho) = \frac{2}{n(n-1)} \sum_{1 \leq i < j \leq n} \rho(X_i, X_j).$$

Define $\theta = \int_{\mathbb{R}^2} \rho(x, y)dF(x)dF(y)$, and $\rho_1(x_1) = \int_{\mathbb{R}} \rho(x_1, x_2)\, dF(x_2)$. Let,

$$h^{(1)}(x_1) = \rho_1(x_1) - \theta, \text{ and,} \tag{1.4}$$

$$h^{(2)}(x_1, x_2) = \rho(x_1, x_2) - h^{(1)}(x_1) - h^{(1)}(x_2) - \theta. \tag{1.5}$$

Then, using Hoeffding's decomposition,

$$U_n(\rho) = \theta + \frac{2}{n}\sum_{i=1}^{n} h^{(1)}(X_i) + \frac{2}{n(n-1)} \sum_{1 \leq i < j \leq n} h^{(2)}(X_i, X_j). \tag{1.6}$$

Dewan and Prakasa Rao (2001) gave a central limit theorem for degenerate and non-degenerate U-statistics based on $\{X_n, n \geq 1\}$ using an orthogonal expansion of the underlying kernel. Dewan and Prakasa Rao (2002) and its corrigendum (Dewan and Prakasa Rao, 2015) obtained a central limit theorem for U-statistics with component-wise monotonic, differentiable, and non-degenerate kernels of degree 2, using Hoeffding's decomposition. The limiting distribution of U-statistics based on $\{X_n, n \geq 1\}$ can also be obtained using the results of Beutner and Zähle (2012) and Beutner and Zähle (2014). The approach of Beutner and Zähle (2012) is based on a modified delta method and quasi-Hadamard differentiability, while Beutner and Zähle (2014) propose a continuous mapping approach. Garg and Dewan (2015) obtained the limiting distribution of U-statistics for $\{X_n, n \geq 1\}$ based on kernels which are functions of Hardy–Krause variation.

Download English Version:

https://daneshyari.com/en/article/5129461

Download Persian Version:

https://daneshyari.com/article/5129461

Daneshyari.com