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Parameter estimation of Markov switching bilinear model using the *(EM)* algorithm

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ABSTRACT

Markov Switching models have known a strong growth since their introduction by James Hamilton in the late 1980's. These models are used as an essential tool for the analysis of the economic cycles. In this paper, we are interested in a class of bilinear models with markov-switching regime ($MS-BL$). These models first appeared in Bibi and Aknouche (2010). Parameter estimation via maximum likelihood (ML) of the ($MS-BL$) model has been considered in Bibi and Ghazel (2015). However, construction and numerical maximization in the approach proposed by Bibi and Ghazel (2015) are computationally intractable. Hence, we propose an expectation–maximization (EM) procedure that provides an alternative method for maximizing the likelihood function in such situations. Convergence and consistency of the (EM) algorithm are discussed in this context. Finally, a Monte Carlo study is presented and two real data examples are proposed.

1. Introduction

Several heteroskedastic models have been developed to capture sudden burst in volatility and extreme values. One of the first models proposed for such purpose in time series analysis is the standard bilinear model proposed by Subba Rao (1978) and Granger and Andersen (1978). Bilinear processes (BL) are a class of non-linear models that has received heightened attention in the probabilistic and statistical literature from their first release. Indeed, they have been widely used for modeling time series $(X_t)_{t\in\mathbb{Z}}$ with occasional sharp spikes which are often found in meteorology, oceanography, geology, biology and agriculture (see Subba Rao (1981) and the references therein for further discussion).

However, a striking feature of these models is that their parametric structure is assumed to be constant in all the sample and cannot incorporate more fundamental changes in the observed series and structural breaks in the dynamic behavior of the data. A natural generalization of the (BL) models mitigates this deficiency and leads to models whose coefficients may vary over time. Markov-switching bilinear model ($MS-BL$) is one of the most versatile models to use when there is consecutive but recurrent regime shifts intercepted by short phases of calm. The $MS-BL$ model is also the simplest extension of the markov-switching ARMA model proposed by Francq and Zakoïan (2001) that allows to capture well changes in the variance over time and lagged error dynamic. It first appeared in Bibi and Aknouche (2010) where necessary and sufficient conditions ensuring the existence of stationary (strict and weak) and ergodic solutions have been given as well as necessary and sufficient conditions of existence of higher order moments. Later, parameter estimation via maximum likelihood (ML) approach has been considered in Bibi and Ghazel (2015). Finally, as supported by authors in Bibi and Ghazel (2015) the $MS-BL$ process can be considered as a second-order approximation to any underlying non-linear process, such as

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standard bilinear models (Granger and Andersen, 1978), Hidden-Markov models (Francq and Roussignol, 1997), $MS-ARMA$ models (Francq and Zakoïan, 2001), some classes of $MS-GARCH$ models (Abramson and Cohen, 2007) and independent-switching bilinear models (Aknouche and Rabehi, 2010). Hence, $MS-BL$ model is a suitable model for estimating non-linearity in mean in a large and a very general context.

In this paper, we are interested in the parameter estimation of $MS-BL$ model. The estimation procedure proposed in Bibi and Ghazel (2015) is not applicable in practice. Indeed, the likelihood function of the available observed (incomplete) data is complicated in structure resulting in enormous difficulties to solve maximization problems. An associated complete-data problem with the same parameters can be formulated from which it is possible to find the maximum likelihood estimations (MLE) in a simpler manner. The expectation–maximization (EM) algorithm (Demster et al., 1977) provides an attractive alternative method for deriving the MLE of an incomplete-data problem by introducing an associated complete-data one. Previous applications of this algorithm to Markov-switching models include works of Kiefer Nicholas (1980) that considered the case of i.i.d. switching regressions, Baum et al. (1970) that considered a scalar system with no explanatory variables or autoregressive dynamics, but with an unobserved Markov switching process for the mean and variance, Liporace (1982) that discussed the vector case and Hamilton (1990) that considered the general systems where the processes are subject to discrete shifts in autoregressive parameters, with the shifts themselves modeled as the outcome of discrete valued Markov process. Applications related to (nonlinear) time series combined with MS models include MS generalized autoregressive heteroskedasticity ($MS\ GARCH$) models introduced by authors in Lamoureux and Lastrapes (1990) that have justified such a combination. Several other authors have taken up the ($MS\ GARCH$) model and studied it in details. They all faced a challenging task when computing the exact likelihood that was infeasible in practice. Some of them have chosen a modified version of the ($MS\ GARCH$) model that overcomes the path dependence problem by maximum likelihood (Haas et al., 2004; Klaassen, 2002). A generalized method of moments (GMM) procedure and a Bayesian Markov chain Monte Carlo (MCMC) algorithm have been proposed as alternative estimation methods, not dependent on the likelihood, by authors in Francq and Zakoïan (2001); Bauwens et al. (2011) and Bauwens et al. (2010). However, Author in Augustyniak (2014) has proposed to compute the MLE of the $MS\ GARCH$ model without resorting to a simplification of the model by developing an approach based on the Monte Carlo EM algorithm. He also showed how the asymptotic variance–covariance matrix of the MLE can be estimated while in Francq and Zakoïan (2001) it was not possible to obtain the asymptotic standard errors of the GMM estimates due to numerical difficulties. Hence, we deeply believe that making bilinear models more flexible by introducing Markov switching is a worthwhile endeavor since at least many $MS\ GARCH$ models can be expressed as a $MS\ BL$ model with the advantage that the nonnegativity constraints on the parameters is relaxed by this latter.

The paper is organized as follows: Section 2 presents the $MS-BL$ model and recalls some of its statistical properties. In Section 3, we describe our estimation methodology and in Section 4 a simulation study is performed. Two real data examples are proposed in Section 5 to illustrate the usefulness of the proposed methodology. Finally, concluding remarks are given in Section 6.

2. The MS-BL model and its statistical properties

A real valued time series $(X_t)_{t\in\mathbb{Z}}$, $\mathbb{Z}=\{\ldots,-1,0,1,\ldots\}$, defined on a probability space $(\Omega,\Im,P)$ is said to be a general Markov Switching Bilinear ($MS\ BL_d$) time series if it admits the representation:

$$X_t=\sum_{i=1}^{p}a_i(s_t)X_{t-i}+\sum_{j=1}^{q}b_j(s_t)e_{t-j}+\sum_{i=1}^{P}\sum_{j=1}^{Q}c_{ij}(s_t)X_{t-i}e_{t-j}+e_t \tag{2.1}$$

where $(e_t)_{t\in\mathbb{Z}}$ is an i.i.d sequence of random variables with zero mean and variance 1. $(s_t)_{t\in\mathbb{Z}}$ is a first order Markov chain on a finite state space $\mathbb{S}=\{1,\ldots,d\}$, stationary, homogeneous, irreducible and independent of $(e_t)_{t\in\mathbb{Z}}$ and of lagged X_t i.e. s_t and $\{(e_t,X_{k-1}),k\le t\}$ are independent. For all $i,j\in\mathbb{S}$ the transition probability matrix $\mathbf{P}=\left(p_{ij}\right)_{1\le i,j\le d}$, that determines the evolution in s_t is given by $p_{ij}=P(s_t=j\,|s_{t-1}=i)$ with $\sum_{j=1}^{d}p_{ij}=1$. To understand the typical behavior of $MS-BL$ models, we generated a multitude of trajectories. Figs. 1 and 2 are respectively a synthetic trajectory generated from a $MS-BL(1)$ model, for $a(1)=0.09$, $a(2)=0.4033$, $p_{11}=0.65$, $p_{22}=0.85$ and $\sigma^2=1$, and the corresponding autocorrelation function ACF. Fig. 1 illustrates what is new in $MS-BL$ model against the non regime switching bilinear model (i.e., the classical BL model). Mean reverting effect and the presence of occasional but recurrent consecutives spikes are common properties of standard BL models and $MS-BL$ models. However, the BL model presents lengthy periods of calm followed by brief sequence of high intensity spikes while in the $MS-BL$ model there is short periods of calm followed by moderate intensity spikes. Most of the generated trajectories had the same behavior. Furthermore, from Fig. 2 one can observe a cyclical behavior of the synthetic data generated by the $MS-BL$ model against no cyclical effect for the standard BL model represented in Fig. 1.

As in the next section the exact likelihood function will be replaced by the likelihood conditional on the first p_0 observations, where $p_0=\max\{p,q\}+1$, the probability law governing the initial unobserved state s_{p_0} is drawn from a separate probability distribution whose parameter $\rho=(\rho_1,\ldots,\rho_d)$ is a $(d\times 1)$ vector unrelated to $\mathbf{P}$:

$$\rho_k=P\left(s_{p_0}=k\setminus\Phi_1(p_0),\underline{\theta}\right),\ k=1,\ldots,d,$$