Statistics and Probability Letters 132 (2018) 28–34

Contents lists available at ScienceDirect

Statistics and Probability Letters

journal homepage: www.elsevier.com/locate/stapro

Weak convergence of the linear rank statistics under strong mixing conditions

Lucia Tabacu

Old Dominion University, United States

A R T I C L E I N F O

Article history:
Received 30 January 2017
Received in revised form 9 July 2017
Accepted 2 September 2017
Available online 21 September 2017

Keywords:
Rank statistics
Strong mixing
Weak convergence
U-statistics
p-continuous

A B S T R A C T

We obtain the asymptotic distribution of the linear rank statistics under weak dependence. We consider a sequence of strong mixing random vectors with unequal dimensions and show the asymptotic normality of the rank statistics based on overall ranking.

© 2017 Elsevier B.V. All rights reserved.

1. Introduction

In this paper we study the weak convergence of linear rank statistics defined using a sequence of strong mixing random vectors. The vector coordinates can be dependent or independent and have varying dimensions. We give the technical conditions under which the linear rank statistics verify the asymptotic normal distribution. The model that we invoke here follows as in Brunner and Denker (1994). In Brunner and Denker (1994) the authors obtained the asymptotic normality of the linear rank statistics where the observations form an array of independent random vectors and also discussed multiple applications to factorial designs with repeated measurements.

Other papers that deal with linear rank statistics in the context of weak dependence are Denker and Rösler (1985), where the classical two sample linear rank statistic for stationary uniform mixing and strong mixing is considered, Fears and Mehra (1974), Ahmad and Lin (1980) that extend the Chernoff–Savage theorem. Harel and Puri (1993) studied the weak convergence for linear rank statistics for nonstationary random variables under ϕ-mixing and strong mixing assumptions.

One of the goals of this paper is to update the weak convergence result for rank statistics with recent development in the study of weakly dependent processes. The method of proof is based on Taylor expansion of the linear rank statistic in terms that have a U-statistic structure. Then we use a result by Dehling and Wendler (2010) to find the second moment estimators for the terms in the decomposition and the Lindeberg result by Utev (1990).

2. Simple linear rank statistics

We use the standard notation introduced in Brunner and Denker (1994) and in Denker and Tabacu (2015).

Let $\mathbf{X}_i = (X_{i1}, \ldots, X_{im_i})$, $i \geq 1$ be a sequence of random vectors with continuous marginal distributions $F_{ij}(x) = P(X_{ij} \leq x)$, $x \in \mathbb{R}$, $j = 1, \ldots, m_i$. Let $N(n) = \sum_{i=1}^{n} m_i$ be the number of observations involved in the vectors $\mathbf{X}_1, \ldots, \mathbf{X}_n$ and

E-mail address: ltabacu@odu.edu.

http://dx.doi.org/10.1016/j.spl.2017.09.001
0167-7152/© 2017 Elsevier B.V. All rights reserved.

$\lambda_{ij}^{(n)}$ $(1 \leq j \leq m_i,\ 1 \leq i \leq n)$ be known regression constants. We assume that

$$\max_{1\leq i\leq n,\ 1\leq j\leq m_i} |\lambda_{ij}^{(n)}| = 1. \tag{2.1}$$

Define

$$H^{(i)}(x) = \sum_{j=1}^{m_i} F_{ij}(x), \qquad \widehat{H}^{(i)}(x) = \sum_{j=1}^{m_i} \mathbb{I}(X_{ij} \leq x), \tag{2.2}$$

$$F^{(i,n)}(x) = \sum_{j=1}^{m_i} \lambda_{ij}^{(n)} F_{ij}(x), \qquad \widehat{F}^{(i,n)}(x) = \sum_{j=1}^{m_i} \lambda_{ij}^{(n)} \mathbb{I}(X_{ij} \leq x), \tag{2.3}$$

$$H_n(x) = \frac{1}{N(n)} \sum_{i=1}^{n} H^{(i)}(x), \qquad \widehat{H}_n(x) = \frac{1}{N(n)} \sum_{i=1}^{n} \widehat{H}^{(i)}(x), \tag{2.4}$$

$$F_n(x) = \frac{1}{N(n)} \sum_{i=1}^{n} F^{(i,n)}(x), \qquad \widehat{F}_n(x) = \frac{1}{N(n)} \sum_{i=1}^{n} \widehat{F}^{(i,n)}(x), \tag{2.5}$$

where $\mathbb{I}(A)$ is the indicator function of the set A. The simple linear rank statistic that we are interested in is defined by

$$L_n(J) = \int_{-\infty}^{\infty} J\left(\frac{N(n)}{N(n)+1}\widehat{H}_n\right) d\widehat{F}_n = \frac{1}{N(n)} \sum_{i=1}^{n} \sum_{j=1}^{m_i} \lambda_{ij}^{(n)} J\left(\frac{R_{ij}(n)}{N(n)+1}\right), \tag{2.6}$$

where $R_{ij}(n)$ denotes the rank of X_{ij} among all random variables $\{X_{kl} : 1 \leq k \leq n,\ 1 \leq l \leq m_k\}$ and $J : (0, 1) \to \mathbb{R}$ denotes an absolutely continuous score function. Let

$$T_n(J) = L_n(J) - \int_{-\infty}^{\infty} J(H_n) dF_n, \tag{2.7}$$

$$B_n(J) = \int_{-\infty}^{\infty} J(H_n) d(\widehat{F}_n - F_n) + \int_{-\infty}^{\infty} J'(H_n)(\widehat{H}_n - H_n) dF_n, \tag{2.8}$$

$$\sigma_n^2(J) = N(n)^2 \mathrm{Var}(B_n(J)). \tag{2.9}$$

We assume that the underlying random vectors satisfy the strong mixing assumption. Recall that $(\mathbf{X}_n)_{n\geq 1}$ satisfies the strong mixing condition if

$$\alpha(n) = \sup_{k\in\mathbb{N}} \sup_{A\in\mathcal{F}_1^k, B\in\mathcal{F}_{k+n}^{\infty}} |P(A \cap B) - P(A)P(B)| \to 0 \text{ as } n \to \infty,$$

where $\mathcal{F}_a^b = \sigma(\mathbf{X}_i, a \leq i \leq b)$ is the σ-field generated by the random vectors $\mathbf{X}_i$, $a \leq i \leq b$.

We also need the concept of p-continuity of a function. The definition of p-continuity was introduced in Borovkova et al. (2001) and it is related to the continuity property given by Denker and Keller (1986). Definition 2.1 is adapted to a nonstationary sequence of random variables and a function that is not symmetric. Examples of p-continuous kernels are given in Borovkova et al. (2001).

Definition 2.1. Let (X, Y) be a $\mathbb{R}^2$-valued random variable and $g : \mathbb{R}^2 \to \mathbb{R}$ be a measurable function. Then we say that g is p-continuous (with respect to (X, Y)) if there exists a function $\phi : (0, \infty) \to (0, \infty)$ with $\phi(\epsilon) \to 0$ as $\epsilon \to 0$ such that for all $\epsilon > 0$

$$E(|g(X', Y') - g(X'', Y')|^p \mathbb{I}(|X' - X''| \leq \epsilon)) \leq \phi(\epsilon),$$

$$E(|g(X', Y') - g(X', Y'')|^p \mathbb{I}(|Y' - Y''| \leq \epsilon)) \leq \phi(\epsilon)$$

hold for all random variables X', X'', Y', Y'' such that (X', Y') has the distribution $P_{X,Y}$ or $P_X \times P_Y$, X'' has the same distribution as X and Y'' has the same distribution as Y.

3. Asymptotic normality

The weak convergence of $\frac{N(n)}{\sigma_n(J)} T_n(J)$ under strong mixing assumptions on the random vectors $(\mathbf{X}_i)_{i\geq 1}$ is obtained in Theorem 3.1. Its proof has the same structure as in Brunner and Denker (1994).

Download English Version:

https://daneshyari.com/en/article/5129627

Download Persian Version:

https://daneshyari.com/article/5129627

Daneshyari.com