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Condensation and symmetry-breaking in the zero-range process with weak site disorder

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Abstract

Condensation phenomena in particle systems typically occur as one of two distinct types: either as a *spontaneous* symmetry breaking in a homogeneous system, in which particle interactions enforce condensation in a randomly located site, or as an *explicit* symmetry breaking in a system with background disorder, in which particles condensate in the site of extremal disorder. In this paper we confirm a recent conjecture by Godrèche and Luck by showing, for a zero range process with weak site disorder, that there exists a phase where condensation occurs with an intermediate type of symmetry-breaking, in which particles condensate in a site randomly chosen from a range of sites favoured by disorder. We show that this type of condensation is characterised by the occurrence of a Gamma distribution in the law of the disorder at the condensation site. We further investigate fluctuations of the condensate size and confirm a phase diagram, again conjectured by Godrèche and Luck, showing the existence of phases with normal and anomalous fluctuations.

Keywords: Condensation; Symmetry-breaking; Extended condensate; Interacting particle system; Zero-range process; Disordered system; Weak disorder; Grand canonical ensemble; Extremes; Gamma distribution; Central limit theorem

1. Motivation and background

The purpose of this paper is two-fold. The *first* purpose is to show that for certain low-dimensional particle systems far from equilibrium the simultaneous presence of inter-particle

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interactions and interactions of particles with a spatial disorder can lead to a novel form of symmetry breaking, occurring in a phase when the two competing particle forces are of comparable strength. In these systems we observe that, when the particle density exceeds a certain threshold value, the excess fraction of the particles condensates in a single site. This site is neither chosen uniformly at random (as would be the case in systems with spontaneous symmetry breaking) nor as a function of the underlying site disorder (as would be the case in systems with explicit symmetry breaking) but by a nontrivial random mechanism favouring sites with more extreme site disorder. The existence of such systems was predicted in a recent paper by Godrèche and Luck [11]. The *second* purpose of this paper is to give a further example of the ubiquity of the Gamma distribution in particle systems with condensation, which was first observed in Dereich and Mörters [5]. In our context the Gamma distribution occurs as the universal distribution of the disorder at the condensation site.

The interacting particle model under consideration here is the *zero-range process*, first introduced in the mathematical literature by Spitzer in [16]. The zero-range process has gained importance in the statistical mechanics literature, for example as a generic model for domain wall dynamics in a system far from equilibrium [14] or as a model for granular flow [7,4]. It is also a particularly simple model undergoing a condensation transition, and widely studied for this reason alone [12,8,2]. It is related to the ideal Bose gas and to spatial permutations [6]. The zero-range process has also been studied in a disordered medium, both in infinite [1] and finite [9] geometries, and the latter situation is also the context of the present paper.

Our version of the zero-range process is a continuous time Markov process, which can be described as a system of m indistinguishable particles each located in one of n different sites. Every site can hold an arbitrary number of particles. At each time instance particles move independently given the particle configuration, and the rate at which particles hop from position i to a different position j is given as $r_{ij}u_k$, where k is the number of particles at site i. Here $R = (r_{ij} \colon 1 \leq i, j \leq n)$ is a Q-matrix (i.e. off-diagonal entries are nonnegative and each row sums to zero) describing the unconstrained particle motion, and $(u_k \colon k \geq 0)$ is a sequence of nonnegative weights with $u_0 = 0$, that describes the particle interactions. The term zero-range process comes from the fact that, at any given time instance, the interaction is only between particles in the same site or, in other words, the jump rate above depends on the global particle configuration only through the number k of particles on the site of departure. The case $u_k = k$ corresponds to independent movement of the particles without interaction, but our interest here is mainly in sublinear sequences $(u_k \colon k \geq 0)$, in which particles move slower if they are aggregated at a site with many other particles. One such case would be that $u_k = 1$, for all $k > 0$, meaning that at every site only one particle is free to move. The phenomena of interest in this paper occur when u_k is given as a small perturbation of this case.

Assuming that the finite state Markov chain described above is irreducible, general theory insures that the state of the zero-range process converges in law, as time goes to infinity, to a unique stationary distribution, or steady state. Denoting by Q_i the number of particles located in site i this distribution is explicitly given by

$$P\big(Q_1 = q_1, \ldots, Q_n = q_n\big) = \frac{1}{Z_{m,n}} \prod_{i=1}^{n} \pi_i^{q_i} p_{q_i} \quad \text{if } q_i \geq 0 \text{ are integers with } \sum_{i=1}^{n} q_i = m,$$

where $(\pi_i \colon 1 \leq i \leq n)$ is a positive left eigenvector of R for the eigenvalue zero, $(p_k \colon k \geq 0)$ are derived from $(u_k \colon k \geq 0)$ by $p_0 = 1$ and $p_k = 1/u_1 \cdots u_k$, for $k \geq 1$, and $Z_{m,n}$ is the normalisation constant, or partition function. The most studied case is that of spatial