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Superprocesses with interaction and immigration

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Abstract

We construct a class of superprocesses with interactive branching, immigration mechanisms, and spatial motion. It arises as the limit of a sequence of interacting branching particle systems with immigration, which generalizes a result of Méléard and Roelly (1993) established for a superprocess with interactive spatial motion. The uniqueness in law of the superprocess is established under certain conditions using the pathwise uniqueness of an SPDE satisfied by its corresponding distribution function process. This generalizes the recent work of Mytnik and Xiong (2015), where the result for a super-Brownian motion with interactive immigration mechanisms was obtained. An extended Yamada–Watanabe argument is used in the proving of pathwise uniqueness.

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1. Introduction

Superprocesses, which are also called Dawson–Watanabe processes, are mathematical models from biology and physics and have been studied by many authors; see [1,7,17] and the references therein. They were introduced to describe the asymptotic behaviour of populations

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undergoing random reproduction and spatial migration which are independent of the entire population. Typical examples of the models are biological populations in isolated regions, families of neutrons in nuclear reactions, cosmic ray showers and so on. For the case that the reproduction and migration mechanisms depend on the entire population as well as on individual's position (called as branching interacting particle system (BIPS)), it was first proved in [11] that a sequence of BIPSs converges to a superprocess with interactive spatial motion and binary branching mechanism as the branching rate tends to infinity and the mass of each particle tends to zero.

If we consider a situation where there are some additional sources of population from which immigration occurs during the evolution, we need to consider superprocesses with immigration; see [6,7,10] and the references therein. It is natural to imagine that there are some additional sources immigrating to BIPS. Thus our *first purpose* here is to establish an existence theorem of such a superprocess with interactive branching, immigration mechanisms, and spatial motion (briefly denoted as SIBIM). When we turn to the uniqueness in law of the superprocess, we generalize the work of Mytnik and Xiong [13], where the result for a super-Brownian motion with interactive immigration mechanisms was established. We show that the uniqueness of the solution to the martingale problem for a SIBIM holds under certain conditions, which is the *second purpose* of this work.

To continue with the introduction we present some notation. Given a topological space V, let $\mathscr{B}(V)$ denote the Borel σ-algebra on V. Let $B(V)$ be the set of bounded measurable functions on V. Let $T > 0$ and $D([0, T]; V)$ denote the space of cádlág paths from $[0, T]$ to V furnished with the Skorokhod topology. For a metric space $\bar{V}$ let $P(\bar{V})$ be the family of Borel probability measures on $\bar{V}$ equipped with the Prohorov metric. Let $B(\mathbb{R})$ be furnished with the supremum norm $\|\cdot\|$. We use $C(\mathbb{R})$ to denote the subset of $B(\mathbb{R})$ of bounded continuous functions. For any integer $n \geq 1$ let $C^n(\mathbb{R})$ be the subset of $C(\mathbb{R})$ of functions with bounded continuous derivatives up to the nth order. Let $C_0^n(\mathbb{R})$ denote the space of functions in $C^n(\mathbb{R})$ vanishing at infinity. Let $C_c^n(\mathbb{R})$ be the subset of $C^n(\mathbb{R})$ of functions with compact supports. We use the superscript "+" to denote the subsets of positive elements of the function spaces, e.g., $B(\mathbb{R})^+$. For $f, g \in \mathscr{B}(\mathbb{R})$ write $\langle f, g\rangle = \int_{\mathbb{R}} f(x)g(x)dx$ whenever it exists.

Let $M(\mathbb{R})$ be the space of finite Borel measures on $\mathbb{R}$ equipped with the weak convergence topology. For $\mu \in M(\mathbb{R})$ and $f \in B(\mathbb{R})$ write $\langle \mu, f\rangle = \int f d\mu$. Let $D(\mathbb{R})$ be the set of bounded right-continuous increasing functions f on $\mathbb{R}$ satisfying $f(-\infty) := \lim_{x\to-\infty} f(x) = 0$. We write $f(\infty)$ for $\lim_{x\to\infty} f(x)$ as $f \in D(\mathbb{R})$. Then there is a 1–1 correspondence between $D(\mathbb{R})$ and $M(\mathbb{R})$ assigning a measure to its distribution function. We endow $D(\mathbb{R})$ with the topology induced by this correspondence from the weak convergence topology of $M(\mathbb{R})$. Then for any $M(\mathbb{R})$-valued stochastic process $\{X_t : t \geq 0\}$, its *distribution function process* $\{Y_t : t \geq 0\}$ is a $D(\mathbb{R})$-valued stochastic process. In this paper we always use C_0 to denote a positive constant whose value might change from line to line. Let ∇ and Δ denote the first and the second order spatial differential operators, respectively.

In the present paper we always assume that $(A(\mu))_{\mu\in M(\mathbb{R})}$, a family of the generators of Feller semigroups on $C(\mathbb{R})$, has the following properties:

- All of their domains contain a vector space $\mathcal{D}$ which is independent of μ and dense in $C_0(\mathbb{R})$.
- All constant functions belong to $\mathcal{D}$, and $A(\mu)1 = 0$ for all $\mu \in M(\mathbb{R})$.
- For each $f \in \mathcal{D}$, there is a constant C so that $\|A(\mu)f\| \leq C$ for all $\mu \in M(\mathbb{R})$.
- For each $f \in \mathcal{D}$, the mapping $\mu \mapsto \langle \mu, A(\mu)f\rangle$ is continuous.