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Formal and physical equivalence in two cases in contemporary quantum physics

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ABSTRACT

The application of analytic continuation in quantum field theory (QFT) is juxtaposed to T-duality and mirror symmetry in string theory. Analytic continuation—a mathematical transformation that takes the time variable *t* to negative imaginary time—*it*—was initially used as a mathematical technique for solving perturbative Feynman diagrams, and was subsequently the basis for the Euclidean approaches within mainstream QFT (e.g., Wilsonian renormalization group methods, lattice gauge theories) and the Euclidean field theory program for rigorously constructing non-perturbative models of interacting QFTs. A crucial difference between theories related by duality transformations and those related by analytic continuation is that the former are judged to be physically equivalent while the latter are regarded as physically inequivalent. There are other similarities between the two cases that make comparing and contrasting them a useful exercise for clarifying the type of argument that is needed to support the conclusion that dual theories are physically equivalent. In particular, T-duality and analytic continuation in QFT share the criterion for *predictive equivalence* that two theories agree on the complete set of expectation values and the mass spectra and the criterion for *formal equivalence* that there is a "translation manual" between the physically significant algebras of observables and sets of states in the two theories. The analytic continuation case study illustrates how predictive and formal equivalence are compatible with physical inequivalence, but not in the manner of standard underdetermination cases. Arguments for the physical equivalence of dual theories must cite considerations beyond predictive and formal equivalence. The analytic continuation case study is an instance of the strategy of developing a physical theory by extending the formal or mathematical equivalence with another physical theory as far as possible. That this strategy has resulted in developments in pure mathematics as well as theoretical physics is another feature that this case study has in common with dualities in string theory.

1. Introduction

There is a canonical set of examples of dualities in mathematics and physics which bear some family resemblance. Taxonomies of the dualities related to string theory are laid out in the introduction to this issue and Polchinski (2017). A common feature of dualities in string theory is that the consensus judgment on their physical interpretation is that dual theories that are apparently physically distinct are actually not physically distinct. That is, dual theories are regarded as not merely empirically equivalent, but physically equivalent. I take it that a project for philosophers is to extract the arguments that support this conclusion, to clearly articulate them, and to then evaluate them. While the details of the arguments will differ in important ways from case to case, there are two key generic ingredients that enter into most of these arguments. One is that dual theories agree on the transition amplitudes and mass spectra. Second, the duality transformations supply a "translation manual" between physical descriptions afforded by the dual theories.

The requirement that two theories agree on the mass spectra and the transition amplitudes is a necessary condition for empirical equivalence that string theory inherits from QFT. This paper will examine an example of a mathematical transformation within QFT that also yields a "translation manual," the second key generic ingredient: the application of analytic continuation in QFT. But—crucially—the

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analytic continuation case study differs from dualities in string theory in that theories related by analytic continuation are not judged to be physically equivalent. This example from QFT simpliciter will be compared and contrasted to examples of dualities in string theoretic extensions of QFT. The goal is to contribute to the preliminary stage of the philosophical project by illuminating the sort of argument that could establish the physical equivalence of dual theories. For the most part, the focus will be confined to T-duality and mirror symmetry.

As I will explain in greater detail below, analytic continuation—aka "Wick rotation"—was initially used as a convenient mathematical trick for performing calculations in perturbative QED. Subsequently, the analytic continuation technique was the basis for introducing a more extensive set of mappings between a Euclidean field theory (EFT[1]) and a relativistic QFT. The quantum field theoretic framework in which the relationship between EFT and QFT has been most thoroughly investigated is Wightman's axiomatic QFT. In Wightman QFT, a physical description of a system is given using a set of smeared operator-valued fields and a dense domain of states D_0 in Hilbert space $\mathcal{H}$ on which the fields are defined, including the vacuum state. An EFT model not only agrees with its counterpart QFT model on the QFT expectation values and mass spectra; there is also a complete "translation manual" from physical descriptions in QFT to expressions in EFT (and vice versa). Furthermore, any EFT model satisfying the Osterwalder—Schrader axioms analytically continues to a QFT model satisfying the Wightman axioms. Thus, the type of equivalence that obtains between a QFT and its analytically continued counterpart EFT goes well beyond empirical equivalence, but falls short of physical equivalence. This type of equivalence will be given the label *formal equivalence*.

The moral of the QFT–EFT case study is that one can push empirical and formal equivalence awfully far in a context in which it is a foregone conclusion that theories are physically inequivalent. This promises to be a fruitful strategy for better understanding both analytic continuation and dualities in string theory because the case studies share a number of additional common features. Most notably, they share some features which have been thought to be characteristic of dualities in string theory. Rickles characterizes dualities as a type of symmetry which differ from, for example, gauge symmetries in acting on a space of theories rather than a space of states or configurations within a single theory (2011, 55). Analytic continuation also relates entire theories rather than states or configurations of a single theory. Furthermore, both analytic continuation and the identification of dualities were strategies for theory development. Both produced surprising conclusions. Schwinger, one of the early proponents of Euclidean approaches to QFT, recalled much later that "I well recall the reception this received, running the gamut from 'It's wrong' to 'It's trivial.' (Quoted in Mehra & Milton, 2000, 303). Since Minkowski spacetime is a characteristic feature of special relativity, it was not expected that a theory on Euclidean space or spacetime could be formally equivalent to relativistic QFT. Euclidean approaches have proven to be a fruitful strategy for developing the theoretical framework of QFT. Examples of successes include Wilsonian renormalization group methods, lattice gauge theories, and the Euclidean program for rigorously constructing non-perturbative models of interacting QFTs. Dualities in string theory often carry the pragmatic advantage that calculations which are extremely difficult or impossible without recourse to the duality become easier or at least possible when use is made of the duality (Polchinski 2017; Matsubara, 2013, 486–487). Analytic continuation carries the same pragmatic advantage. The indirect approach of first constructing a Euclidean model and then analytically continuing to a relativistic QFT model is technically more tractable than the approach of directly constructing a QFT. This application of analytic continuation is not undermined by the physical inequivalence between counterpart QFTs and EFTs. Quantum field theorists recognize that—while transition amplitudes, spectra, and certain formal properties are preserved by analytic continuation—the physical interpretation of a model of EFT does not carry over to the corresponding model of QFT.

While the exploratory strategy pursued in this paper bears some resemblance to the compare-and-contrast approaches adopted in other articles in this issue and in the extant philosophical literature on string theory, it is worth pointing out two differences at the outset. Dualities raise some issues that are familiar to philosophers from the analysis of symmetries in general and gauge transformations in particular and from underdetermination of theory by empirical data, inviting a comparison. Rickles (2017) considers gauge transformations not only close analogues to dualities, but dualities to be a species of gauge transformation. Huggett (2017) argues that, in the case of T-duality, dual theories supply equivalent physical descriptions in the same way that theories on Newtonian spacetime that differ only in the choice of inertial reference frame for absolute space supply equivalent physical descriptions (i.e., when it is recognized that absolute space is an artefact of the theoretical description with no referent in the world). Another class of cases to which dualities have been compared are cases in which two "theories" adopt the same formalism, but adopt different physical interpretations of the formalism. Eleanor Knox has compared dual theories to the holonomy and fibre-bundle interpretations of the Aharanov–Bohm effect and the ordinary space and configuration space interpretations of quantum mechanics. One way in which analytic continuation in QFT differs from these other comparator cases is that the gap between empirical and physical equivalence is not produced by physical descriptions that are only partially constrained by the empirical quantities. The analytic continuation case study is different in kind from standard cases of underdetermination. Second, QFT employs the same criterion for empirical equivalence as string theoretic extensions of QFT: agreement on the set of expectation values and mass spectra. The analytic continuation case study in QFT supplies an opportunity to probe the implications that a shared set of transition amplitudes and spectra hold for theoretical equivalence in the relatively simple context of QFT (i.e., relative to string theoretic extensions of QFT). One way of stating the main point made in this paper is that the condition for empirical equivalence that is routinely imposed in QFT can be read as a formal condition on elements of the mathematical formalism (i.e., expectation values, analytic structure). Because this condition is purely formal, it is possible for it to be satisfied by two systems that share the empirically relevant formal structure, but which are physically inequivalent in virtue of having different physical interpretations. To mark this difference from the empirical equivalence criteria typically considered in philosophy of science, the QFT criterion will be labelled *predictive equivalence*. In order to be a stepping stone to physical equivalence, predictive equivalence must be supplemented by a physical interpretation of its elements. Thus, to support the interpretation of dual theories as physically equivalent, predictive equivalence requires a supporting physical interpretation.

The aim of this paper is to identify methodological and interpretive similarities and differences between the cases of analytic continuation in QFT and dualities in string theory. For this purpose, I will treat the two cases as entirely separate and independent developments in physics. However, it should be noted that there are also important physical connections between the use of analytic continuation as a mathematical technique for constructing mathematically consistent models of QFTs and string theory which are not the subject of this paper. String theory and QFT share a common subject matter: particle

[1] Note that in this paper "EFT" stands for Euclidean field theory, not effective field theory, which is also abbreviated as EFT in the QFT and string theory literature.