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Talk about toy models

Joshua Luczak

Institute of Philosophy, Leibniz Universität Hannover, Germany

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ABSTRACT

Scientific models are frequently discussed in philosophy of science. A great deal of the discussion is centred on approximation, idealisation, and on how these models achieve their representational function. Despite the importance, distinct nature, and high presence of toy models, they have received little attention from philosophers. This paper hopes to remedy this situation. It aims to elevate the status of toy models: by distinguishing them from approximations and idealisations, by highlighting and elaborating on several ways the Kac ring, a simple statistical mechanical model, is used as a toy model, and by explaining why toy models can be used to successfully carry out important work without performing a representational function.

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1. Introduction

Scientific models are frequently discussed in philosophy of science. A great deal of the discussion is centred on approximation, idealisation, and on how these models achieve their representational function. Some philosophers have tried to get clear on approximations and idealisations.[1] Some dissect them, typically idealisation, into types.[2] Some have highlighted and discussed their differences.[3] Some have discussed their representational functions and ideals.[4] Many have discussed how these models achieve their representational function.[5] Some have questioned and discussed what these models can tell us about reality.[6] Some have questioned and discussed their role in scientific explanations.[7] The list goes on.

Toy models are easily confused with approximations and idealisations. They are, however, distinct. Unlike idealisations and approximations, toy models do not perform a representational function. That is, they do not represent anything. They are nonetheless important for science. Toy models are models that are not intended to perform a representational function, but rather to perform one or more of the following functions:

1. To learn to use, or to become comfortable with, certain formal techniques (e.g. renormalization). That is, as a pedagogical device.
2. To elucidate certain ideas relevant to a theory. That is, to reach a clearer understanding of an idea, its implications, and its relation to other ideas within a theory.
3. To test the compatibility of various concepts (i.e. in a consistency proof).
4. To generate hypotheses about other systems.

One commonly finds authors using simple models to perform one or more of these functions in the introductory chapters of physics textbooks. When they do so, and do not intend for them to perform a representational function, it is appropriate to regard their models as toy models. Some examples common to statistical mechanics include: Mark Kac's ring model, Paul and Tatyana Ehrenfests' urn (dog-flea) and wind-tree models, the baker's transformation, the Ising model, and the Arnold cat map.

Despite the importance, distinct nature, and presence of toy models, they have received little attention from philosophers.[8]

E-mail address: joshmluczak@gmail.com

[1] See, for example, Achinstein (1968), Bunge (1973), and Laymon (1990).

[2] See, for example, Frigg & Hartmann (2012, Sec. 1.1), McMullin (1985), and Weisberg (2007).

[3] See Norton (2012), for example.

[4] See Weisberg (2007), for example.

[5] See, for example, Aronson, Harré, & Way (1995), Bailer-Jones (2003), Bartels (2006), Frigg & Hartmann (2012) (and the references therein), Giere (2010), Godfrey-Smith (2006), Mundy (1986), Pincock (2012), Suárez (2015), Suppes (2002), Swoyer (1991), and Weisberg (2013).

[6] See Frigg & Hartmann (2012, Sec. 3) and Swoyer (1991), for example.

[7] See, for example, Bokulich (2009, 2011, 2012), Cartwright (1983, Ch. 8), Elgin & Sober (2002), Frigg & Hartmann (2012, Sec. 5.4), and Woodward (2003).

[8] Michael Weisberg (2013, Ch. 7.3) has discussed, what he calls, targetless models. Targetless models are like toy models in that they are not intended to perform a representational function. It is unclear, however, from Weisberg's short discussion of targetless models, whether they are similar to toy models in other respects—such as how and why they can be used to carry out important work.

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Perhaps this is because many philosophers and scientists mistakenly think of them as either a kind of approximation or idealisation. Or perhaps it is because many of the models used in the ways listed above are, on other occasions, used to represent other systems, and this obscures the distinction between these model types. Whatever the case may be, it would be beneficial for our understanding of scientific models to more deeply explore toy models and to engage in focused discussions of them. This paper hopes to advance this goal. It aims to elevate the status of toy models and to encourage more focused discussions of them. This will be achieved by distinguishing them from approximations and idealisations, by highlighting and elaborating on several ways the Kac ring is used as a toy model, and by explaining why it can be successfully used in these ways without performing a representational function. This paper will focus on ways in which the Kac ring can be used to successfully perform functions **2-4**, since the claim that it can be used to successfully achieve **1**, without performing a representational function, is uncontroversial. In speaking by way of the Kac ring model, this paper intends to support the claim that toy models play an important role in science, despite them not performing a representational function.

The next section notes some standard claims made about approximations, idealisations, and scientific representation. These are noted so as to distinguish approximations and idealisations from toy models. Parts of the third section draw on the work of Gottwald & Oliver (2009, Sec. 3). The section begins by highlighting some of the Kac ring's features. The model is then used to elucidate two important statistical mechanical ideas: the reversibility objection and the recurrence objection. Its ability to successfully perform this task without performing a representational function is also discussed in this section. Discussion of the recurrence and reversibility objections encourages using the Kac ring in other ways: as part of a consistency proof and to suggest interesting things about other systems, including real systems. The fourth section includes the consistency proof and a discussion of why an agent can use the model to successfully perform these tasks, despite it not performing a representational function. The fifth section highlights and continues to discuss its use in generating hypotheses about other systems without performing a representational function. The paper ends with a few concluding remarks and with some suggestions about the direction of future work.

2. Approximations, idealisations, scientific representations, and toy models

Approximations and idealisations are frequently discussed in the literature on modelling. Despite there being disagreement about how to precisely characterise these model types, there are certainly some things that can be said about them that are uncontroversial. An approximate model inexactly represents a target system. Typically this is some aspect of the world. Idealised models also represent target systems. These models, however, introduce some kind of deliberate simplification or distortion. These modifications are usually introduced to make it easier to deal with the target. Importantly, approximations and idealisations perform a representational function.

While it is a subject of debate within philosophy of science as to what exactly constitutes a model's representation of a target, there are good reasons to think that the representation can neither simply be reduced to a similarity relation that holds between the model and the target nor to some kind of morphism relation (e.g. isomorphism) that holds between the structures that are instantiated by both the model and the target.[9] On the positive side, it seems fair to say that a model performs a representational function only if its user intends for it to perform a representational function. This fits with a promising and growing view that scientific representation is a practice performed by intentional agents.[10] As Ronald Giere (2004, p. 747) explains, scientists use models to represent portions of the world for various purposes. But, as he continues, it is not the model that is doing the representing; it is the scientist using the model who is doing the representing. If we embrace both of these views of scientific representation, as we will in the remainder of this paper, then we can maintain that models can be used as toy models and that these models do not perform a representational function. That is, that an agent can use a model to successfully perform one or more of the functions **1-4** without intending that it perform a representational function and that in these circumstances these models do not perform a representational function. The next few sections intend to highlight these facts, and to explain why they are so, for functions **2-4**.

Models, such as the Kac ring, can be used by agents to do a lot of interesting and important work simply because either they instantiate certain properties (see the discussions of functions **2–3**) or because they instantiate certain properties that are also known to be instantiated by other systems (see the discussion of function **4**). In the latter type of case, these similarities permit treating the model as an analogue. Moreover, these similarities permit, and are sufficient for, analogical reasoning.[11] That is, they permit, and are sufficient for, employing some version of the following argument schema, where S is some model and T is some other system:

1. S is similar to T in certain (known) respects.
2. S has some further feature Q.
3. Therefore, T also has the feature Q, or some feature Q^* similar to Q.

Importantly, however, as Mauricio Suárez (2015) has argued, by drawing on Nelson Goodman (1968) argument against resemblance theories of artistic representation, similarity is not sufficient for representation.[12] Similarity, as it is typically understood, is a reflexive and symmetric relation. Scientific representations, on the other hand, do not have these logical properties. If similarity were sufficient for representation then, for example, a dilute gas would be a scientific representation of a billiard ball model and itself. But it is neither of these things, so similarity is not sufficient for scientific representation. So then, even if a model instantiates properties that are also instantiated by other systems, this does not entail that it represents any or all of those systems, or anything at all.

3. The Kac Ring

The Kac ring first appeared in a series of lectures given by Mark Kac in1959 at the University of Colorado. The purpose of these lectures was to furnish an introduction to probability theory and its applications to an audience that had little knowledge of these subjects.[13] In a lecture on classical statistical mechanics, Kac (1959, p. 99) used the ring model to introduce his audience to the

[9] See Suárez (2015), Suárez (2003) for more on this point. And see Suárez (2015) for a state-of-the-art review of the philosophical literature on scientific representation.

[10] See Suárez (2015).

[11] See Bartha (2013) for a comprehensive discussion of analogies and analogical reasoning.

[12] See also Suárez (2003).

[13] See the forward and preface to Kac (1959).