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Maximum likelihood localization: When does it fail?☆

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Abstract

Maximum likelihood is a criterion often used to derive localization algorithms. In particular, in this paper we focus on a distance-based algorithm for the localization of nodes in static wireless networks. Assuming that Ultra Wide Band (UWB) signals are used for inter-node communications, we investigate the ill-conditioning of the Two-Stage Maximum-Likelihood (TSML) Time of Arrival (ToA) localization algorithm as the Anchor Nodes (ANs) positions change. We analytically derive novel lower and upper bounds for the localization error and we evaluate them in some localization scenarios as functions of the ANs' positions. We show that particular ANs' configurations intrinsically lead to ill-conditioning of the localization problem, making the TSML-ToA inapplicable. For comparison purposes, we also show, through some examples, that a Particle Swarm Optimization (PSO)-based algorithm guarantees accurate positioning also when the localization problem embedded in the TSML-ToA algorithm is ill-conditioned.

Keywords: Two-Stage Maximum-Likelihood (TSML) algorithms; Time of Arrival (ToA); Localization

1. Introduction

Nowadays, wireless indoor localization is an interesting topic in many applications [1]. Indoor positioning systems aim at providing precise position estimates inside buildings, which is a particularly tricky task, due to phenomena such as non-line-of-sight and multipath, caused by walls and obstacles. In particular, time-based positioning techniques rely on inter-nodes distance estimates evaluated from the times of flight of signals traveling between pairs of nodes. Given the pair-wise distance estimates between a few nodes, denoted as Anchor Nodes (ANs), and a Target Node (TN), the TN's position can be estimated [2]. Among the wide variety of localization algorithms which have been proposed in the literature, in this paper we focus on the Two-Stage Maximum-Likelihood (TSML) Time of Arrival (ToA) algorithm proposed in [3]. This is a well-known algorithm, based on inter-node distance estimates, which yields a closed-form position estimate and can attain the Cramer–Rao Bound [2]. Unfortunately, despite its quasi-optimality, depending on the nodes' relative positions the localization problem "embedded" in the TSML-ToA algorithm can become ill-conditioned, leading to far inaccurate position estimates. This is detrimental in practical applications (e.g., industrial localization), where ANs may not be freely positioned.

In this paper, we investigate the limits of maximum likelihood-based localization techniques. More precisely, we first derive novel lower and upper bounds for the distance between the true TN position and its estimate, i.e., the localization error. For comparison, we investigate the localization accuracy of a Particle Swarm Optimization (PSO)-based localization algorithm. It will be shown that the PSO allows accurate localization even in those scenarios where the TSML-ToA algorithm fails.

This paper is organized as follows. In Section 2, novel lower and upper bounds for the TN localization error are analytically derived. In Section 3, the values of these bounds are evaluated in a few illustrative scenarios. Section 4 concludes the paper.

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2. Problem localization conditioning

We assume to know the positions of three ANs, denoted as $\{\underline{s}_i = [x_i, y_i]^T\}_{i=1}^3$ and we aim at localizing a TN with coordinates $\underline{u} = [x, y]^T$. In the following, without leading the generality of the derivation, we assume that $x \neq 0$ and $y \neq 0$. Given the three exact distances $\{r_i\}_{i=1}^3 = \{\|\underline{s}_i - \underline{u}\|\}_{i=1}^3$, the TN position could be determined by simply intersecting the three circumferences centered in $\{\underline{s}_i\}_{i=1}^3$ with radii $\{r_i\}_{i=1}^3$, respectively. In the following, the TSML-ToA algorithm is first briefly outlined and, then, lower and upper bounds for the positioning error are derived.

2.1. The TSML-ToA algorithm

The *first phase* of the TSML-ToA algorithm involves the solution of the system of equations:

$$\underline{\underline{G}}\,\underline{\omega} = \underline{h} \tag{1}$$

where: $\underline{\omega} = [x, y, n]$, $n \triangleq \|\underline{u}\|^2$,

$$\underline{\underline{G}} \triangleq \begin{pmatrix} x_1 & y_1 & -0.5 \\ x_2 & y_2 & -0.5 \\ x_3 & y_3 & -0.5 \end{pmatrix} \qquad \underline{h} \triangleq \frac{1}{2}\begin{pmatrix} K_1 - r_1^2 \\ K_2 - r_2^2 \\ K_3 - r_3^2 \end{pmatrix} \tag{2}$$

and $\{K_i\}_{i=1}^3 \triangleq \{\|s_i\|^2\}_{i=1}^3$. Observe that $\underline{\underline{G}}$ is ill-conditioned when: (i) the three ANs lie nearly on the same line (corresponding to two columns nearly linearly dependent); (ii) at least two ANs are very close (namely, two rows are similar).

Since the true distance measurements $\{r_i\}_{i=1}^3$ are not available, one can only rely on their noisy estimates, which will be denoted as

$$\hat{r}_i \triangleq r_i + \delta r_i \quad i \in \{1, 2, 3\} \tag{3}$$

where $\{\delta r_i\}_{i=1}^3$ are the estimation errors. Hence, instead of (1), one is left with

$$\underline{\underline{G}}\,\hat{\underline{\omega}} = \hat{\underline{h}} \tag{4}$$

where $\hat{\underline{\omega}} \triangleq \underline{\omega} + \underline{\delta\omega}$ and $\hat{\underline{h}} \triangleq \underline{h} + \underline{\delta h}$. Observe that, from (3), it follows that

$$\underline{\delta h} = -\begin{pmatrix} r_1\delta r_1 + 0.5(\delta r_1)^2 \\ r_2\delta r_2 + 0.5(\delta r_2)^2 \\ r_3\delta r_3 + 0.5(\delta r_3)^2 \end{pmatrix} \simeq -\begin{pmatrix} r_1\delta r_1 \\ r_2\delta r_2 \\ r_3\delta r_3 \end{pmatrix} \tag{5}$$

where the last approximation has been obtained neglecting quadratic perturbations—the approximation holds if the perturbations are sufficiently small.

The *second phase* of the TSML-ToA algorithm is meant to take into account the dependence of n on the other two unknowns of (1) and involves solving:

$$\underline{\underline{G}}'\,\underline{\phi} = \underline{h}' \tag{6}$$

where:

$$\underline{\underline{G}}' \triangleq \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \qquad \underline{\phi} \triangleq \begin{pmatrix} x^2 \\ y^2 \end{pmatrix} \qquad \underline{h}' \triangleq \begin{pmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3 \end{pmatrix}$$

and ω_j, $j \in \{1, 2, 3\}$, denotes the jth component of $\underline{\omega}$. Assuming that only $\hat{\underline{\omega}}$ is known, one is left with

$$\underline{\underline{G}}'\,\hat{\underline{\phi}} = \hat{\underline{h}}' \tag{7}$$

where $\hat{\underline{\phi}} \triangleq \underline{\phi} + \underline{\delta\phi}$, $\hat{\underline{h}}' \triangleq \underline{h}' + \underline{\delta h}'$, and

$$\underline{\delta h}' = \begin{pmatrix} 2\omega_1\delta\omega_1 + (\delta\omega_1)^2 \\ 2\omega_2\delta\omega_2 + (\delta\omega_2)^2 \\ \delta\omega_3 \end{pmatrix} \simeq \begin{pmatrix} 2\omega_1\delta\omega_1 \\ 2\omega_2\delta\omega_2 \\ \delta\omega_3 \end{pmatrix} \tag{8}$$

where $\{\delta\omega_i\}_{i=1}^3$ denote the ith component of $\underline{\delta\omega}$. The last approximation in (8) has been obtained, as in (5), neglecting quadratic perturbations.

The final position estimate $\hat{\underline{u}}$ is $\hat{\underline{u}} = \underline{\underline{U}}\sqrt{\hat{\underline{\phi}}}$ where $\underline{\underline{U}} = \mathrm{diag}(\mathrm{sign}(\hat{\underline{\omega}}_1))$ [3]. Denoting as $\underline{\delta u} = \hat{\underline{u}} - \underline{u}$ the error on the position estimate, we derive lower and upper bounds for the localization error $\|\underline{\delta u}\|$.

2.2. Bounds for position estimation error

First, lower and upper bounds for the norms $\|\underline{\delta\omega}\|$ and $\|\underline{\delta\phi}\|$ of the errors on the solution of (4) and (7), respectively, are derived. The results are then combined together to finally obtain bounds on the norm of the localization error $\|\underline{\delta u}\|$.

From (4) and (1), one obtains $\underline{\underline{G}}\,\underline{\delta\omega} = \underline{\delta h}$ and taking the norm of both sides, it follows:

$$\|\underline{\underline{G}}\,\underline{\delta\omega}\| = \|\underline{\delta h}\| \leq \|\underline{\underline{G}}\|\,\|\underline{\delta\omega}\|. \tag{9}$$

Assuming that $\underline{\underline{G}}$ is not singular, one can conclude that:

$$\|\underline{\delta\omega}\| = \|\underline{\underline{G}}^{-1}\,\underline{\delta h}\| \leq \|\underline{\underline{G}}^{-1}\|\,\|\underline{\delta h}\|. \tag{10}$$

Finally, from (9) and (10), the following bounds for $\|\underline{\delta\omega}\|$ can be derived:

$$\|\underline{\underline{G}}\|^{-1}\|\underline{\delta h}\| \leq \|\underline{\delta\omega}\| \leq \|\underline{\underline{G}}^{-1}\|\,\|\underline{\delta h}\|. \tag{11}$$

From (7) and (6), one obtains $\underline{\underline{G}}'\,\underline{\delta\phi} = \underline{\delta h}'$ and, taking the norm of both sides, one derives:

$$\|\underline{\underline{G}}'\,\underline{\delta\phi}\| = \|\underline{\delta h}'\| \leq \|\underline{\underline{G}}'\|\,\|\underline{\delta\phi}\|. \tag{12}$$

Moreover, defining

$$\underline{\underline{H}} \triangleq \underline{\underline{G}}'^T\underline{\underline{G}}' \qquad \underline{\delta\ell} \triangleq \underline{\underline{G}}'^T\underline{\delta h}' \tag{13}$$

one obtains $\underline{\delta\phi} = \underline{\underline{H}}^{-1}\,\underline{\delta\ell}$ and, hence,

$$\|\underline{\delta\phi}\| \leq \|\underline{\underline{H}}^{-1}\|\,\|\underline{\delta\ell}\|. \tag{14}$$

Finally, from (12) and (14), the following bounds can be derived:

$$\|\underline{\underline{G}}'\|^{-1}\|\underline{\delta h}'\| \leq \|\underline{\delta\phi}\| \leq \|\underline{\underline{H}}^{-1}\|\,\|\underline{\delta\ell}\|. \tag{15}$$

From (13):

$$\underline{\delta\ell} = \underline{\underline{G}}'^T\underline{\delta h}' = \begin{pmatrix} 2\omega_1\delta\omega_1 + \delta\omega_3 \\ 2\omega_2\delta\omega_2 + \delta\omega_3 \end{pmatrix} = \underline{\underline{C}}\,\underline{\delta\omega} \tag{16}$$