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Adaptive algebraic multiscale solver for compressible flow in heterogeneous porous media

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ABSTRACT

This paper presents the development of an Adaptive Algebraic Multiscale Solver for Compressible flow (C-AMS) in heterogeneous porous media. Similar to the recently developed AMS for incompressible (linear) flows (Wang et al., 2014) [19], C-AMS operates by defining primal and dual-coarse blocks on top of the fine-scale grid. These coarse grids facilitate the construction of a conservative (finite volume) coarse-scale system and the computation of local basis functions, respectively. However, unlike the incompressible (elliptic) case, the choice of equations to solve for basis functions in compressible problems is not trivial. Therefore, several basis function formulations (incompressible and compressible, with and without accumulation) are considered in order to construct an efficient multiscale prolongation operator. As for the restriction operator, C-AMS allows for both multiscale finite volume (MSFV) and finite element (MSFE) methods. Finally, in order to resolve high-frequency errors, fine-scale (pre- and post-) smoother stages are employed. In order to reduce computational expense, the C-AMS operators (prolongation, restriction, and smoothers) are updated adaptively. In addition to this, the linear system in the Newton–Raphson loop is infrequently updated. Systematic numerical experiments are performed to determine the effect of the various options, outlined above, on the C-AMS convergence behaviour. An efficient C-AMS strategy for heterogeneous 3D compressible problems is developed based on overall CPU times. Finally, C-AMS is compared against an industrial-grade Algebraic MultiGrid (AMG) solver. Results of this comparison illustrate that the C-AMS is quite efficient as a nonlinear solver, even when iterated to machine accuracy.

1. Introduction

Accurate and efficient simulation of multiphase flow in large-scale heterogeneous natural formations is crucial for a wide range of applications, including hydrocarbon production optimization, risk management of Carbon Capture and Storage, water resource utilizations and geothermal power extractions. Unfortunately, considering the size of the domain along with the high resolution heterogeneity of the geological properties, such numerical simulation is often beyond the computational capacity of traditional reservoir simulators. Therefore, Multiscale Finite Element (MSFE) [1–5] and Finite Volume (MSFV) [6,7] methods and their extensions have been developed to resolve this challenge. A comparison of different multiscale methods,

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based on their original descriptions, has been studied in the literature [8]. MSFV and MSFE methods map a discrete fine-scale system to a much coarser space. In MultiGrid (MG) terminology [9], this map is considered as a special prolongation operator, represented by locally-supported (and adaptively updated) basis functions [10]. The restriction operator is then defined based on either a Finite Element (MSFE), Finite Volume (MSFV), or a combination of both.

MSFV has been applied to a wide range of applications (see, e.g., [10–21]), thus recommending multiscale as a very promising framework for the next-generation reservoir simulators. However, most of these developments, including the state-of-the-art algebraic multiscale formulation (AMS) [19], have focused on the incompressible (linear) flow equations.

When compressibility effects are considered, the pressure equation becomes nonlinear, and its solution requires an iterative procedure involving a parabolic-type linear system of equations [22]. Therefore, the development of an efficient and general algebraic formulation for compressible nonlinear flows is crucial in order to advance the applicability of multiscale methods towards more realistic problems.

The present study introduces the first algebraic multiscale iterative solver for compressible flows in heterogeneous porous media (C-AMS), along with a thorough study of its computational efficiency (CPU time) and convergence behaviour (number of iterations).

In contrast to cases with incompressible flows, the construction of basis functions for compressible flow problems is not straightforward. In the past, incompressible elliptic [23], compressible elliptic [10,24], and pressure-independent parabolic [25] basis functions have been considered. However, the literature lacks a systematic study to reveal the benefit of using one option over the other, especially when combined with a fine-scale smoother stage. Moreover, no study of the overall efficiency of the multiscale methods (based on the CPU time measurements) has been done so far for compressible three-dimensional problems.

In order to develop an efficient prolongation operator, in this work, several formulations for basis functions are considered. These basis functions differ from each other in the amount of compressibility involved in their formulation, ranging from incompressible elliptic to compressible parabolic types. In terms of the restriction operator, both MSFE and MSFV are considered, along with the possibility of mixing iterations of the former with those of the latter, allowing C-AMS to benefit from the Symmetric Positive Definite (SPD) property of MSFE and the conservative physically correct solutions of MSFV.

The low-frequency errors are resolved in the global (multiscale) stage of C-AMS, while high-frequency errors are tackled using a second-stage smoother at fine-scale. In this paper, we consider two options for the smoothing stage: the widely used local correction functions with different types of compressibility involved (i.e., more general than the specific pressure-independent operator [25]), as well as ILU(0) [26]. The best C-AMS procedure is determined among these various strategies, on the basis of the CPU time for 3D heterogeneous problems. It is important to note that the setup and linear system population are measured alongside the solve time – a study which has so far not appeared in the previously published compressible multiscale works.

Though C-AMS is a conservative method (i.e., only a few iterations are enough in order to obtain a high-quality approximation of the fine-scale solution), in the benchmark studies of this work, it is iterated until machine accuracy is reached. And, thus, its performance as an exact solver is compared against an industrial-grade Algebraic MultiGrid (AMG) method, SAMG [27]. This comparative study for compressible problems is the first of its kind, and is made possible through the presented algebraic formulation, which allows for easy integration of C-AMS in existing advanced simulation platforms. Numerical results, presented for a wide range of heterogeneous 3D domains, illustrate that the C-AMS is quite efficient for simulation of nonlinear compressible flow problems.

The paper is structured as follows. First, the Compressible Algebraic Multiscale Solver (C-AMS) is presented, where several options for the prolongation, restriction operators as well as the second-stage solver are considered. Then the adaptive updating of the C-AMS operators are studied, along with the possibility of infrequent linear system updates in the Newton–Raphson loop. Numerical results are subsequently presented for a wide range of 3D heterogeneous test cases, aimed at determining the optimum strategy. Finally, the C-AMS is compared with an Algebraic MultiGrid Solver (i.e., SAMG) both in terms of the number of iterations and overall CPU time.

2. Compressible flow in heterogeneous porous media

Single phase compressible flow in porous media, using Darcy's law (without gravity and capillary effects), can be stated as:

$$\frac{\partial}{\partial t}(\phi\rho) - \nabla \cdot \left(\rho\, \boldsymbol{\lambda} \cdot \nabla p\right) = \rho q, \tag{1}$$

where ϕ, ρ, and q are the porosity, density, and the source terms, respectively. Moreover, $\boldsymbol{\lambda} = \boldsymbol{K}/\mu$ is the fluid mobility with positive-definite permeability tensor, $\boldsymbol{K}$, while μ is the fluid viscosity.

The semi-discrete form of this nonlinear flow equation using implicit (Euler-backward) time integration reads

$$\frac{\phi^{n+1}}{\Delta t} - \frac{\phi^n \rho^n}{\Delta t\, \rho^{n+1}} - \frac{1}{\rho^{n+1}} \nabla \cdot \left(\rho^{n+1} \boldsymbol{\lambda} \cdot \nabla p^{n+1}\right) = q, \tag{2}$$

which is linearized as