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Test Method

Determination of dynamic properties of flax fibres reinforced laminate using vibration measurements

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ABSTRACT

Experimental and numerical methods to identify the linear viscoelastic properties of flax fibre reinforced polymer (FFRP) composite are presented in this study. The method relies on the evolution of storage modulus and loss factor as observed through the frequency response. Free-free symmetrically guided beams were excited in the dynamic range of 10 Hz to 4 kHz with a swept sine excitation focused around their first modes. A fractional derivative Zener model has been identified to predict the complex moduli. A modified ply constitutive law has been then implemented in a classical laminates theory calculation (CLT) routine. Overall, the Zener model fitted the experimental results well. The storage modulus was not frequency dependant, while the loss factor increased with frequency and reached a maximum value for a fibre orientation of 70°. The damping of FFRP was, respectively, 5 and 2 times higher than for equivalent carbon and glass fibres reinforced epoxy composites.

1. Introduction

Structural parts of aircraft and land vehicles are submitted to dynamic loading. Excitations coming from the powertrain, road surfaces and aerodynamic flows, cause mechanical vibrations. In order to improve the acoustical and vibrational comfort, natural fibre reinforced polymer composites, exhibiting interesting dissipative properties, can be used. Indeed, compared to conventional composite materials, the damping performance of flax fibre reinforced polymer (FFRP) can be, respectively, twice or three times higher than that of glass or carbon fibre reinforced polymer composites [1,2].

Damping in undamaged composite materials is induced by several microscopic level mechanisms, such as viscoelastic elongation of the matrix and/or fibres, friction between both components at their interfaces. Moreover, in the particular case of flax fibre reinforced polymer composites, the friction between fibres inside bundles represents an additional mechanism of dissipation of energy, which can also increase this phenomenon [3]. At mesoscale level, parameters such as layer orientation, interlaminar effects and vibration coupling can also affect energy dissipation.

In order to assess the damping within materials, vibration techniques have the advantage of rapidly exploring a wide range of frequencies (10 Hz–1 kHz). Thus, many authors have studied the frequency dependence of composite materials [4–7]. Crane et al. [8] have proposed an analytical approach to predict the frequency dependence of laminates properties using classical laminates theory (CLT) in complex domains. Duke et al. [9] have tested several flax fibre reinforced composites with different matrices, and have compared them to glass and carbon fibre reinforced composite. They observed that the damping properties obtained with the FFRP were generally better than those of the carbon and glass fibre reinforced composites.

The frequency dependence is a typical feature of viscoelastic materials. This dependence induces variations of the complex moduli when the frequency of excitation changes. In order to assess these variations, it is necessary to understand the material constitutive equations that relate the stresses to the strains with respect to time or frequency [10], with the help of linear viscoelasticity theory. These relations are expressed by linear differential equations or convolution integrals. They have the main benefit of being expressed both in frequency and time domains.

The present study aims at presenting a method dedicated to the identification of the evolution of complex moduli of FFRP laminates in a large frequency band with the help of a fractional derivative Zener model. This identification has been done thanks to experimental tests on a specific device between 10 Hz and 4 kHz.

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2. Linear viscoelastic model

Materials submitted to mechanical loading store energy by elastic deformation. For purely elastic materials, this energy is totally and immediately returned during the unloading phase. However, in the case of viscoelastic materials, the energy stored is completely returned but with some delay due to inner rearrangements. For such materials, the stress depends on the strain history [10]. Several models describing the behaviour of a viscoelastic material in the structural dynamic domain are available in the literature. Among them, the fractional derivative Zener model (Fig. 1) has been used by many authors to describe the viscoelastic behaviour [11]. This model is mathematically described in Eq. (1), where $\sigma(t), \varepsilon(t), \tau$, α are respectively the stress and strain as a function of time, the relaxation time and the α-order fractional derivative coefficient ($0 < \alpha < 1$). $D^\alpha[\bullet]$ is an α-order fractional derivative operator. The mechanical stiffness E_0 and E_∞ are, respectively, the dynamic modulus at very low frequency and at high frequency. The fractional derivative model is described in Fig. 1.

$$\sigma(t) + \tau^\alpha D^\alpha[\sigma(t)] = E_0\varepsilon(t) + E_\infty\tau^\alpha D^\alpha[\varepsilon(t)] \tag{1}$$

In the case of a steady state harmonic excitation, the complex modulus (E^*) is derived from Eq. (2), where the fractional derivative operator $D^\alpha[\bullet]$ is replaced by a multiplication by $(j\omega)^\alpha$. j is the imaginary unit and $\omega_n = \omega\tau$.

$$E^*(\omega) = \frac{\sigma^*(\omega)}{\varepsilon^*(\omega)} = \frac{E_0 + E_\infty(j\omega_n)^\alpha}{1 + (j\omega_n)^\alpha} \tag{2}$$

Thus, it is possible to write Eq. (2) in the form of Eq. (3), in which $\eta(\omega) = \frac{E''(\omega)}{E\prime(\omega)}$ represents the material's loss factor. The real part $E\prime(\omega)$ of E^*, so called storage modulus, represents the elastic behaviour of the tested material. The imaginary part $E''(\omega)$ or loss modulus represents the viscous or damping behaviour of the material.

$$E^*(\omega) = E\prime(\omega) + jE''(\omega) = E\prime(\omega)[1 + j\eta(\omega)] \tag{3}$$

Using the constitutive equation (Eq. (1)), $E\prime(\omega)$ and $\eta(\omega)$ can be expressed by Eqs. (4) and (5), where $\frac{c = E_\infty}{E_0}$.

$$E\prime(\omega) = E_0\frac{1 + (c+1)\cos\left(\frac{\alpha\pi}{2}\right)\omega_n^\alpha + c\omega_n^{2\alpha}}{1 + 2\cos\left(\frac{\alpha\pi}{2}\right)\omega_n^\alpha + \omega_n^{2\alpha}} \tag{4}$$

$$\eta(\omega) = \frac{(c-1)\sin\left(\frac{\alpha\pi}{2}\right)\omega_n^\alpha}{1 + (c+1)\cos\left(\frac{\alpha\pi}{2}\right)\omega_n^\alpha + c\omega_n^{2\alpha}} \tag{5}$$

3. Classical laminate theory applied to viscoelasticity

With the help of the individual properties of each ply, the CLT allows one to compute the elastic properties of a stack of unidirectional (UD) composite plies (i.e. longitudinal and transverse storage moduli, Poisson's ratios and shear moduli). In order to establish a viscoelastic version of CLT, the so-called correspondence principle [10] has been used. Initially introduced on homogeneous materials, this principle has been extended to heterogeneous and composite materials by Hashin [12]. In the case of a sinusoidal steady-state excitation, it is possible to measure and replace the elastic properties by the appropriate complex ones, i.e. the conversion of the elastic solutions to viscoelastic ones. Thus, the laminate's relations derived from the CLT can be used to predict the overall viscoelastic moduli of the laminate from the elementary ply properties. According to the elastic CLT method, the complex constitutive relation linking forces (N^*) and bending moments (M^*) to strains (ε^*) and membrane curvatures (κ^*) can be expressed by Eq. (6).

$$\begin{bmatrix} N^*(\omega) \\ M^*(\omega) \end{bmatrix} = \begin{bmatrix} A^*(\omega) & B^*(\omega) \\ B^*(\omega) & D^*(\omega) \end{bmatrix} \begin{bmatrix} \varepsilon^*(\omega) \\ \kappa^*(\omega) \end{bmatrix} \tag{6}$$

The complex laminate stiffness matrices $[\mathrm{ABD}]^*$ are given by Eqs. (7)–(9), based on the complex reduced stiffness matrix $[\tilde{Q}^*_{ij}(\omega)]$, where h_n represents the position of the nth ply in the thickness of the laminate and N is the total number of plies (Fig. 2).

$$A^*_{ij} = A'_{ij} + jA''_{ij} = \sum_{n=1}^{N}\left[\tilde{Q}^*_{ij}\right]_n (h_n - h_{n-1}) \tag{7}$$

$$B^*_{ij} = B'_{ij} + jB''_{ij} = \frac{1}{2}\sum_{n=1}^{N}\left[\tilde{Q}^*_{ij}\right]_n \left(h_n^2 - h_{n-1}^2\right) \tag{8}$$

$$D^*_{ij} = D'_{ij} + jD''_{ij} = \frac{1}{3}\sum_{n=1}^{N}\left[\tilde{Q}^*_{ij}\right]_n \left(h_n^3 - h_{n-1}^3\right) \tag{9}$$

The nth layer in which plies are oriented with an angle (α_n) with respect to the loading direction, the coefficients $[\tilde{Q}^*_{ij}]_n$ are obtained from $[Q^*_{11}(\omega)]_n$ using Eqs. (10)–(15), where $p_n = \cos\alpha_n$ and $q_n = \sin\alpha_n$. The $[Q^*_{11}]_n$ are the complex reduced stiffness.

$$\left[\tilde{Q}^*_{11}\right]_n = \left[Q^*_{11}\right]_n p_n^4 + \left[Q^*_{22}\right]_n q_n^4 + 2\left(\left[Q^*_{12}\right]_n + 2\left[Q^*_{66}\right]_n\right)p_n^2 q_n^2 \tag{10}$$

$$\left[\tilde{Q}^*_{12}\right]_n = \left(\left[Q^*_{11}\right]_n + \left[Q^*_{22}\right]_n - 4\left[Q^*_{66}\right]_n\right)p_n^2 q_n^2 + \left[Q^*_{12}\right]_n\left(p_n^4 + q_n^4\right) \tag{11}$$

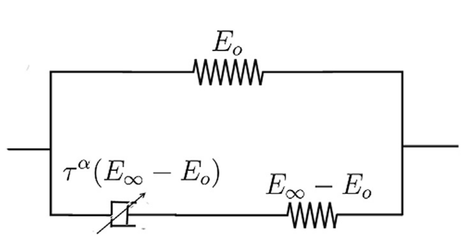

Fig. 1. Fractional derivative Zener model.

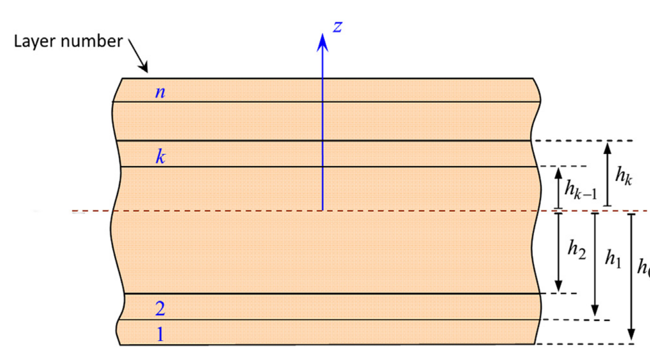

Fig. 2. Laminate's architecture representation [12].