Information Fusion 29 (2016) 142–148

Contents lists available at ScienceDirect

Information Fusion

journal homepage: www.elsevier.com/locate/inffus

A novel algorithm for fuzzy soft set based decision making from multiobserver input parameter data set

José Carlos R. Alcantud*

Facultad de Economía y Empresa, BORDA Research Unit and Multidisciplinary Institute of Enterprise (IME), University of Salamanca, Salamanca 37007, Spain

ARTICLE INFO

Article history:
Available online 1 September 2015

Keywords:
Fuzzy soft set
Resultant fuzzy soft set
Comparison table
Decision making

ABSTRACT

We present two innovations that produce a novel approach to the problem of fuzzy soft set based decision making in the presence of multiobserver input parameter data sets. The first novelty consists of a new process of information fusion that furnishes a more reliable resultant fuzzy soft set from such input data set. The second one concerns the mechanism that decides among the alternatives in this resultant fuzzy soft set. It relies on scores computed from a relative Comparison matrix. The advantages of our novel procedure are a higher power of discrimination and a well-determined final solution.

1. Introduction

Many real life problems require to use imprecise, uncertain or subjective data. Hence their solutions involve the application of mathematical principles that potentially capture these features. Fuzzy set theory caused a profound change in Mathematics by allowing partial membership. Since Zadeh [1] introduced fuzzy sets, a vast literature on their properties and applications to decision making has been produced. For example, Mardani et al. [2] is an extensive analysis of papers about fuzzy multi-criteria decision making published in the period 1994–2014. Tanino [3] or Fodor and Roubens [4] are a short sample of classical references. Furthermore, one of the most used preference structures in group decision making problems under uncertainty is the fuzzy preference relation (cf., Castro et al. [5], which provides an application to consensus-driven group recommender systems).

However in some practical problems, imprecise individual or collective knowledge cannot be faithfully represented by fuzzy sets. This constraint calls for generalizations of this notion and related variations which may supply more suitable models.

In this regard, Atanassov [6,7] proposes the concept of intuitionistic fuzzy sets. New intuitionistic fuzzy multi-attribute group decision making methods are developed e.g., in Chen et al. [8] or Wei [9] among other recent references. The use of interval-valued Atanassov intuitionistic fuzzy sets in multi-expert decision making is exemplified in De Miguel et al. [10]. Bustince and Burillo [11] prove that the concept of vague set coincides with the notion of intuitionistic fuzzy set. Xu and Cai [12] provide a systematic introduction to intuitionistic fuzzy aggregation methods and their many application to decision making.

A direct extension of fuzziness is the general field of hesitancy in fuzzy sets. Hesitant fuzzy sets are introduced by Torra [13]. A reference work is Xu [14] (see also the special issue introduced by Herrera et al. [15], especially the survey Rodríguez et al. [16]). Multiexpert multicriteria decision making under this requirement is explored by Xia et al. [17] or Tan et al. [18] among others.

Given the proliferation of extending notions, it is also important to explore their relationships. Hesitant fuzzy sets can be represented as fuzzy multisets ([13, Lemma 14]) and as type-2 fuzzy sets ([13, Lemma 16]). Bustince et al. [19] is an updated account of types of fuzzy sets and their connections. These authors prove that the original mathematical formulation of an interval type-2 fuzzy set (resp., Atanassov intuitionistic fuzzy set, vague set, grey set, interval-valued fuzzy set, interval-valued Atanassov intuitionistic fuzzy set) corresponds to a set-valued fuzzy set or a hesitant fuzzy set. Set-valued fuzzy sets can be regarded as type-2 fuzzy sets: [19, Section V.A]. Bustince et al. [20] show that fuzzy sets and interval-valued fuzzy sets are particular cases of interval type-2 fuzzy sets. An overview of the mathematical relationships between intuitionistic fuzzy sets and other theories modeling imprecision is given by Deschrijver and Kerre [21].

1.1. Soft sets and extensions

In a different vein, Molodtsov [22] initiates the theory of soft sets. Quoting from Feng and Zhou [23, Section 1], soft set theory "is considered as a new mathematical tool for dealing with uncertainties which is free from the inadequacy of parameter tools. In soft set theory, the

* Tel.: +34923294640.
E-mail address: jcr@usal.es
URL: http://diarium.usal.es/jcr

http://dx.doi.org/10.1016/j.inffus.2015.08.007

problem of setting the membership function simply does not arise as in fuzzy set theory, which makes the theory convenient and easy to use in practice." Indeed, Molodtsov [22] shows its applicability to several fields and establishes some fundamental results subsequently complemented by works like Maji et al. [24] and Aktaş and Çağman [25] among others. Interestingly, Molodtsov [22] shows that the models by fuzzy sets and soft sets are not independent. Other important works regarding the connection among soft sets, fuzzy sets and other soft computing models include Ali [26] and Feng et al. [27,28].

Soft sets have been extended in various ways starting with Maji et al. [29] who introduce fuzzy soft sets. Wang et al. [30] introduce hesitant fuzzy soft sets, which combine the ideas of hesitancy (cf., Torra [13]) with the latter concept. Han et al. [31] and Zou and Xiao [32] are concerned with incomplete soft sets, which may arise from errors in data measurement, errors of data understanding or restrictions in data collection. Feng et al. [33] introduce choice value soft sets in order to improve and further extend Çağman and Enginoğlu's [34] *uni-int* decision making method.

1.2. Fuzzy soft sets and decision making

Let us focus on the context of fuzzy soft set based decision making. Then the researcher must face the fact that there is no universally accepted criterion for evaluating the alternatives. This is the cost to pay for tackling problems whose nature is subjective or humanistic. The pioneering Roy and Maji [35] formulate a solution for an object recognition problem where the recognition strategy relies on multiobserver input parameter data set. Obviously this formulation can be adapted to other choice situations with inputs having the same structure. We intend to improve the performance of their algorithm at the two stages of their proposal, that we proceed to describe.

Stage 1. Roy and Maji [35] propose to begin with an aggregation procedure that yields a single resultant fuzzy soft set from preliminary multi-source information. We show that their original approach, which is universally accepted in this context henceforth, may result into a heavy loss of information that ultimately generates uncertainty. Consequently we argue that it is convenient to use an alternative proposal.

Stage 2. Here we address the pure decision making problem: how do we evaluate the alternatives from the information in the resultant fuzzy soft set?

Roy and Maji [35] propose to construct a Comparison matrix that permits to compute scores for the alternatives.

In order to solve the same problem, a different procedure at Stage 2 is given by Kong et al. [36]. These authors claim that the Roy and Maji's algorithm is incorrect on the grounds of a single naive example. However there is little doubt that such "counterexample" does not support such an extreme view (cf., Feng et al. [37, Section 3.2]).

In addition, Feng et al. [37] point out that the disparity of opinions between [36] and [35] is whether the criterion for making a decision should use scores or fuzzy choice values (understood as the sum of all membership values across attributes). In this controversy we concur with Feng et al.'s argument that the completely redesigned approach by scores in Roy and Maji [35] is more suitable for making decisions in an imprecise environment. As to their own proposal for solving the problem at Stage 2, Feng et al. [37] fully incorporate subjectivity by proposing an adjustable method based on level soft sets. Therefore in their approach the optimal choice is dependent upon the selected level soft sets. Their model introduces potential sources of uncertainty, in the form of threshold fuzzy sets, threshold values, or a choice among decision rules (mid-level, top-level). The practitioner has neither assistance to decide among these possibilities, nor a proper comparative study which supports the idea that their method is more reliable than earlier proposals.

1.3. Contribution and organization of this paper

We formulate another information fusion procedure that overcomes the handicap found at Stage 1. Put shortly, it consists of replacing the 'AND' operator used in [35] (namely, the minimum) by the other prominent example of t-norm in multi-valued logic: namely, the product.

We also propose a novel decision method at Stage 2 of the problem. In line with Roy and Maji's acclaimed proposal it appeals to scores and produces a unique well-determined outcome. But we produce a different Comparison table that eschews the use of incongruous "crisp" values at the core of the definition of Roy and Maji's Comparison matrix. To achieve this goal we evaluate the *relative* differences in membership values across the alternatives.

Due to these innovations, our procedure is considerably less inconclusive than the aforementioned solutions in fuzzy soft set decision making. As is shown by examples from the literature, these procedures tend to produce many ties that are avoided under our position.

This paper is organized as follows. Section 2 recalls some terminology and definitions. Section 3 contains our main contributions. Firstly we present the problem. We discuss the aggregation issue when the input is a set of multiobserver data. Then we propose a novel solution for the problem, and we compare it with previous solutions by exploring examples that have provided arguments in the literature. We conclude in Section 4.

2. Definitions: soft sets and fuzzy soft sets

We adopt the usual description and terminology for soft sets and their extensions: U denotes a universe of objects and E denotes a universal set of parameters.

Definition 1 (Molodtsov [22]). A pair (F, A) is a soft set over U when $A \subseteq E$ and $F : A \longrightarrow \mathcal{P}(U)$, where $\mathcal{P}(U)$ denotes the set of all subsets of U.

A soft set over U is regarded as a parameterized family of subsets of the universe U, the set A being the parameters. For each parameter $e \in A$, $F(e)$ is the subset of U approximated by e or the set of e-approximate elements of the soft set. To put an example, if $U = \{c_1, c_2, c_3, c_4\}$ is a universe of cars and A contains the parameter e that describes "white color" and the parameter e' that describes "diesel engine" then $F(e) = \{c_1\}$ means that the only car with white color is c_1 and $F(e') = \{c_1, c_3\}$ means that the only cars with diesel engine are c_1 and c_3.

Many papers conduct formal investigations of this and related concepts. For example, Maji, Bismas and Roy [24] develop this notion and define among other concepts: soft subsets and supersets, soft equalities, intersections and unions of soft sets, et cetera. Furthermore, Feng and Li [38] give a systematic study on several types of soft subsets and various soft equal relations induced by them. Concerning (pure) soft set based decision making, we refer the reader to Maji et al. [39], Çağman and Enginoğlu's [34] and Feng and Zhou [23].

In order to model more general situations, the following notion is subsequently proposed and investigated in [29]:

Definition 2 (Maji et al. [29]). A pair (F, A) is a fuzzy soft set over U when $A \subseteq E$ and $F : A \longrightarrow \mathbf{FS}(U)$, where $\mathbf{FS}(U)$ denotes the set of all fuzzy sets on U.

Obviously, every soft set can be considered as a fuzzy soft set. Following with our car example above, fuzzy soft sets permit to deal with other properties like "expensive" or "modernly designed" for which partial memberships are natural.

As is well known, when both U and A are finite (as in the application references mentioned above) soft sets and fuzzy soft sets can be represented either by matrices or in tabular form. Rows are attached