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3D reconstruction: Why should the accuracy always be presented in the pixel unit?☆

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ABSTRACT

This paper analyses the error presentation of parallax-based techniques (mainly stereoscopy and structured light). They are usually presented using an absolute (mm) or a relative (%) scale. These results are hard to compare between different systems as they are system-dependent. This paper presents results using the pixel unit which avoids the influence of geometric parameters. Moreover it is apt at evaluating whether the system under-performs or is similar compared to theoretical accuracy.

1. Introduction

The comparison of results produced by different systems is critical. However, in some cases, it is complex to compare results between systems because they rely on parameters that are device specific. In particular, the accuracy of parallax-based techniques (mainly stereoscopy and structured light) is a function of the geometric parameters. However, authors usually present their results using absolute (in millimetres) or relative scale (%) [1,2,3,4,5]. These results are hardly transferable to other systems. This paper presents results using the pixel unit which avoids the influence of any geometric parameters. Moreover it is apt at evaluating it provides a good way to evaluate whether the system under or over-performs compared to theory. The first part presents the calculation of the theoretical error applied to stereoscopy and structured light. Second, a structured light system is implemented to test the methodology and the potential outcomes of the proposed method. Two systems have been tested with the same pattern but different equipment in order to ensure a relevant comparison between both systems.

2. Calculation of the theoretical error

Theoretical calculation of the error allows to evaluate whether the device under or over-performs compared to theory. It ensures that main sources of error are properly mastered. In the case of parallax-based devices, the localisation of the point is limited by the pixel resolution. A high definition image provides much more detail than a low definition image. First, the error has been calculated in the stereoscopy case, second, equations are applied to structured light. All calculations are made in the epipolar plane which contains all deformation (and information). The interested reader may find out more about epipolar geometry in the chapter of the book [6]. If cameras are horizontally aligned, epipolar planes are horizontal and the following case can be generalised to 3D for every height.

2.1. Theoretical error for stereoscopy

A point p is viewed by two cameras 1 and 2 with respectively an angle α and β. The baseline is the distance d see Fig. 1.

The abscissa position x from both devices is given by:

$$\begin{cases} x &= z\ \tan(\alpha) \\ x &= d - (z - z_0)\tan(\beta) \end{cases} . \quad (1)$$

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Usually, cameras are placed next to each other, therefore, z_0 is small compared to z. If x is withdrawn and z_0 is neglected compared to z, the error is given by proceeding with the differential:

$$\left|\frac{\delta z}{z}\right| = \left|(\delta(\tan(\alpha)) + \delta(\tan(\beta)))\frac{z}{d}\right| \tag{2}$$

where:

- $\frac{\delta z}{z}$ is the relative error at working depth z;
- d is the baseline;
- α and β are respectively the angle between the observed point and the camera 1 straight axis and the same for camera 2;
- $\delta\tan(\alpha) = \frac{\delta x_{p_1}}{f_{p_1}}$, where x_{p_1} is the pixel position in camera 1 of the object, δx_{p_1} is the pixel error and f_{p_1} is the focal length of camera 1 expressed in pixels.
- $\delta\tan(\beta) = \frac{\delta x_{p_2}}{f_{p_2}}$, where x_{p_2} is the pixel position in camera 2 of the object, δx_{p_2} is the pixel error and f_{p_2} is the focal length of camera 2 expressed in pixel.

Eq. (2) may be rewritten as in Eq. (3). This equation gives the *relative* error for a stereoscopic device.

$$\frac{\delta z}{z} = \left(\frac{\delta x_{p_1}}{f_{p_1}} + \frac{\delta x_{p_2}}{f_{p_2}}\right)\frac{z}{d}. \tag{3}$$

Or, as the pixel error is given for the complete device: $\delta x_{p_1} = \delta x_{p_2} = \delta x_p$. To evaluate the pixel accuracy, $\delta x_p = 1$. Eq. (3) may be written as given in Eq. (4).

$$\frac{\delta z}{z} = \left(\frac{1}{f_{p_1}} + \frac{1}{f_{p_2}}\right)\frac{z\delta x_p}{d}. \tag{4}$$

The absolute error is easily given by:

$$\delta z = \left(\frac{1}{f_{p_1}} + \frac{1}{f_{p_2}}\right)\frac{z^2\delta x_p}{d}. \tag{5}$$

Eqs. (4) and (5) aim at evaluating the error in case of bad positioning of one pixel. In these cases, $\delta p_x = 1$ and, the error in the pixel unit is given by:

$$\delta x_p = \left(\frac{f_{p_1}f_{p_2}}{f_{p_1} + f_{p_2}}\right)\frac{d\delta z}{z^2}. \tag{6}$$

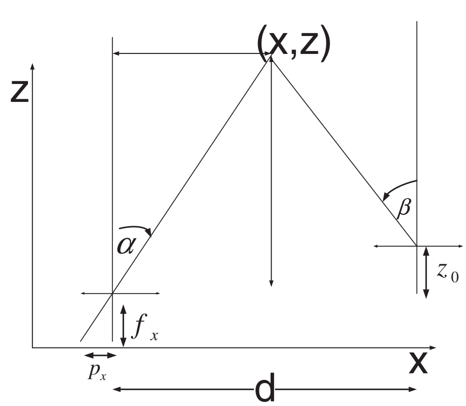

Fig. 1. Illustration of the parameters used for the positioning error of the point P at (x, z), α and β are the angles between the camera central line and the point of interest and d is the baseline, the distance z_0 is the z position of the right camera. The focal length f_x and the pixel position p_x is used for the conversion in pixel.

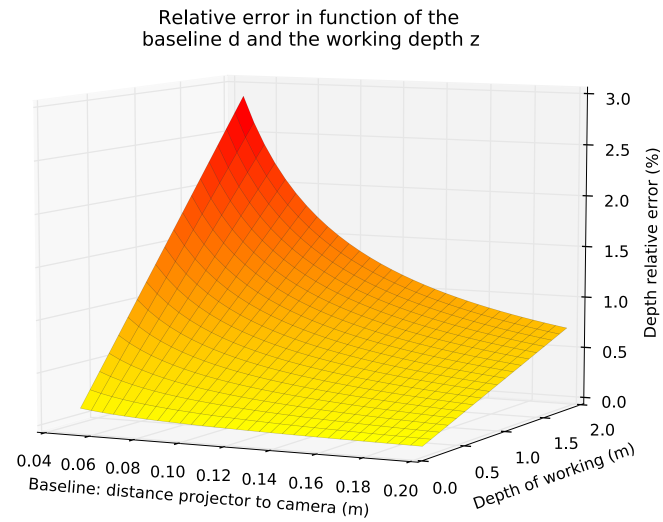

Fig. 2. Theoretical relative error of a 3D reconstruction for a structured light system where $f_x = 1311$.

The latter equation provides the result in the pixel unit. It offers the advantage that, contrary to the relative and the absolute error, it does not depend on the geometry of the system. Results are therefore easier to compare and it becomes obvious to predict what the absolute error will be according to a certain system.

2.2. Theoretical error for structured light

In structured light, a camera is replaced with a projector. Equations are identical but for one aspect: usually, the projector could be calibrated to reduce the error due to the projector. Therefore, $\tan\beta$ could be determined with good accuracy and $\delta\tan\beta \approx 0$. Similarly to stereoscopy, δx_p could be set to 1 to obtain the error due to bad positioning of 1 pixel. The relative error given by Eq. (2) becomes:

$$\left|\frac{\delta z}{z}\right| = \left|\frac{z\delta x_p}{df_x}\right|. \tag{7}$$

The relative error is plotted in Fig. 2. It illustrates the relative error which is directly proportional to the working depth z and proportional to the inverse of the baseline d. Therefore, the error is expected to be constant if multiplied by $\frac{d}{z}$. However, this would provide results hard to interpret, that is why the pixel unit is used to give meaning to the results.

The absolute error is given by Eq. (8).

$$\delta z = \frac{z^2\delta x_p}{df_x} \tag{8}$$

And the error in pixel is given by Eq. (9).

$$\delta x_p = \frac{f_x d\delta z}{z^2} \tag{9}$$

3. Material and method

Two systems have been implemented to illustrate the presentation of the results. The first is a test bench and the second device is an endoscopic one.