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Orthogonal Polar V Transforms and application to shape retrieval ☆

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ABSTRACT

The traditional orthogonal moments (e.g., Zernike moments) are formulated with polynomials as their basis that often face the problem of computation difficulty especially with the high-order moments. In this paper, we present a novel set of transforms namely the Polar V Transforms (PVTs). We can use the PVTs not only to generate the rotation-invariant features but also to capture global and local information of images. Since the PVTs basis functions can keep a low order of polynomials, we can significantly speed-up the runtime for computing the kernels. The experimental results have demonstrated that our proposed method outperforms the previous methods in runtimes and achieves very good results in shape retrieval compared to the previous methods especially when the images with high degree of perspective distortions.

1. Introduction

The features of image shapes play an important role in human recognition and perception. The image shape descriptors can be categorized into two groups: contour and region-based descriptors. The contour-based shape descriptors exploit only the features of the shape boundaries but ignore the potentially important features of shape interior. Early work includes Fourier descriptors [17], Wavelet descriptors [1,5], and the Curvature scale space method [15]. On the other hand, the region-based shape descriptors consider both of the boundaries and interior regions of the object shape. As the most commonly used approaches for region-based shape descriptors, moments have been utilized as pattern features in many applications, including image analysis [22,18,23], pattern recognition [9,8], texture classification [24], object indexing [16,25,11] and image matching [2].

The moments theory provides useful series expansions to represent an object shape. The general image moment can be defined by a kernel (also as known as a basis function) $\psi_{nm}(x,y)$, and an image intensity function $f(x,y)$, which is shown below:

$$M_{nm} = \iint_{\Omega} \psi_{nm}(x,y) f(x,y)\,\mathrm{d}x\,\mathrm{d}y. \quad (1)$$

From the mathematical point of view, moments are projections of the function $f(x,y)$ onto a basis set of $\psi_{nm}(x,y)$. For a geometric moment, the basis set is a monomial set of $\{x^n y^m\}$. Since a monomial set is not orthogonal, it will be very sensitive to noise, wide dynamic range, and a large amount of redundant information.

The orthogonal basis functions based moments, e.g., the Legendre and Zernike polynomials, were first introduced by Teague [22] to represent an image by a set of mutually independent descriptors with a minimal amount of information redundancy. One of the main advantages of using the moments is easy to construct the rotation invariants. An efficient way is to transform a rotation into a shift in the polar coordinates. In the case of complex moments, this shift may cause a phase change that can be eliminated by the multiplication of proper moments, such as the Zernike moments (ZMs) [22], the pseudo-Zernike moments (PZMs) [23], and the Orthogonal Fourier–Mellin moments (OFMMs) [19].

One major disadvantage of using these moments is the high computational complexity due to the nature of monomials/polynomials formulation especially the high-order moments. Several approaches [12,3] have been proposed to reduce the computational complexity. They observed that in some cases moments can be evaluated by utilizing the recurrent relations of the kernel polynomials and thus speed-up the computation process. However, there are two main issues: (1) even with this speed-up it still cannot achieve the real-time computation, and (2) the numerical errors will be increased due to the use of recursive formulas.

In this paper, we present a novel set of transforms namely Polar V Transforms (PVTs). We can use the PVTs not only to generate the rotation-invariant features but also to capture global and local information of images. Differ from the polynomial radial basis

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functions in the traditional orthogonal rotation-invariant moments (ORIMs), the PVTs are based on a class of orthogonal piecewise linear polynomials which is named as the weighted V-system. The basis functions of PVTs can keep the order of polynomials on a very low constant (degree one in PVTs). Thus, the computation of the PVTs is much simpler without performing the computations of high order polynomials that can drastically speed-up the computation time. In addition, the weighted V-system is a class of wavelet function set in which the PVTs can also maintain the multi-scale features of images. This unique feature is very useful for the pattern analysis of images.

The rest of the paper is organized as below. Section 2 reviews the construction of ORIMs. Section 3 introduces the weighted V-system. In Section 4, we describe the orthogonal Polar V Transforms. Section 5 presents the experimental results. Finally, in the Section 6 we give concluding remarks.

2. Review of the orthogonal rotation-invariant moments (ORIMs)

The orthogonal rotation-invariant moments (ORIMs) are constructed in the form of

$$M_{nm} = c_{nm} \iint_{\Omega} f(r,\theta) z_{nm}^{*}(r,\theta) r \, dr \, d\theta, \tag{2}$$

where $f(r,\theta)$ is an image defined on a continuous unit disk $\Omega = \{(r,\theta) | 0 \leqslant r \leqslant 1, 0 \leqslant \theta \leqslant 2\pi\}$, c_{nm} is a normalizing factor, $z_{nm}(r,\theta)$ is the ORIMs basis functions of order n and repetition m, which is constructed as product of a certain radial polynomial part and an angular phase component, i.e.,

$$z_{nm}(r,\theta) = R_{nm}(r) e^{jm\theta}. \tag{3}$$

Because rotating an image would not change the ORIMs magnitude, the magnitudes of ORIMs $\|M_{nm}\|$ have been used as a pattern feature in many applications [9,8,25,19,10,6].

Furthermore, the ORIMs basis functions are orthogonal to each other on the unit disk, i.e.,

$$\iint_{\Omega} z_{nm}^{*}(r,\theta) z_{pq}(r,\theta) r dr d\theta = \frac{1}{c_{nm}} \delta_{np} \delta_{mq}, \tag{4}$$

where $\delta_{np} = 1$ if $n = p$, and 0 otherwise.

The ORIMs basis functions differ in the radial polynomial. There are a few sets of ORIMs with their kernels complying with the form of (2), namely, Zernike moments (ZMs), with their radial kernels defined as [22]:

$$R_{nm}(r) = \sum_{i=0}^{\frac{n-|m|}{2}} \frac{(-1)^{i} (n-i)!}{i! \left(\frac{n+|m|}{2} - i\right)! \left(\frac{n-|m|}{2} - i\right)!} r^{n-2i}. \tag{5}$$

pseudo-Zernike moments (PZMs) [23], a variation of ZMs, with their radial kernels defined as:

$$R_{nm}(r) = \sum_{i=0}^{n-|m|} \frac{(-1)^{i} (2n+1-i)!}{i!(n+|m|-i)!(n-|m|+1-i)!} r^{n-i}, \tag{6}$$

and orthogonal Fourier–Mellin moments (OFMMs) [19]:

$$R_{n}(r) = \sum_{i=0}^{n} \frac{(n+i+1)!}{i!(n-i)!(i+1)!} r^{i} \tag{7}$$

Fig. 1(a) and (b) show the graphs of the first 10 radial polynomials of ZMs and OFFMs respectively.

We observe that the radial polynomials of ZMs, PZMs and OFMMs involve a number of factorial terms, which are inherently faced with the problem of computation difficulty, especially with the increasing of the degrees of polynomials.

3. The weighted V-system

3.1. Overview

The V system of degree $k (k = 0, 1, 2, \ldots)$ is a complete orthogonal function system consisting of a series of piecewise polynomials. Song et al. [20,21] has applied it to study the similarity of geometric models. In this section, we give an overview of the V-system. The detailed descriptions can be found in [20,21,7].

The V-system of degree k consists of two classes of functions. The first class is the set of the first $k+1$ normalized Legendre polynomials on the interval $[0,1]$ that denoted as $\{V_{k,1}^{i}(x)\}_{i=1}^{k+1}$, and the function generator of $G_k = \{V_{k,2}^{i}(x)\}_{i=1}^{k+1}$. The second class of V system is the set of functions generated by the multiscale squeezing, shift and duplication of G_k defined as

$$V_{k,n}^{i,j}(x) = \begin{cases} \sqrt{2^{n-2}} V_{k,2}^{i}\left(2^{n-2}\left(x - \frac{j-1}{2^{n-2}}\right)\right), & x \in \left(\frac{j-1}{2^{n-2}}, \frac{j}{2^{n-2}}\right); \\ 0, & \textit{otherwise}, \end{cases}$$

for $n = 3, 4, \ldots,$ $i = 1, 2, \ldots, k+1$ and $j = 1, 2, \ldots, 2^{n-2}$. Consequently the family of functions $\{V_{k,1}^{i}\} \cup \{V_{k,2}^{i}\} \cup \{V_{k,n}^{i,j}\}$ is called the V system of degree k.

For simplicity, we denote the basis functions of V system as $\{v_n(x), n = 1, 2, \ldots\}$ throughout this paper.

3.2. The weighted V-system

The starting point of this paper is to replacing the traditional high-degree radial polynomials by piecewise polynomials with low-degree polynomials. The V-system introduced in Section 3.1 is normalized orthogonal on the interval $[0,1]$, i.e., $\int_0^1 v_n(x) v_m(x) dx = \delta_{nm}$. Each basis function in the V-system of degree k is a piecewise k degree polynomial. We observe that the degree of polynomials can be fixed on a small number if we apply the V-system to an orthogonal transformation. This can greatly reduce the computational complexity. However, the V-system does not keep orthogonality in polar coordinate, i.e. $\int_0^1 v_n(r) v_m(r) r dr \neq \delta_{nm}$. Therefore, The V-system cannot be used for constructing the ORIMs directly. To solve this problem, we have applied the Gram–Schmidt orthogonalization algorithm (as shown in Algorithm 1) to transform the original V-system into a weighting V-system. In Algorithm 1, we denote the nth basis function as $v_n(r)$ and $wv_n(r)$, $(n = 1, 2, \ldots.)$ respectively.

Algorithm 1. The weighted Gram–Schmidt orthogonalization

Require: $v_n(r), \ n = 1, 2, \ldots$
Ensure: $wv_n(r), n = 1, 2, \ldots$
$m \leftarrow \int_0^1 v_1(r) v_1(r) r dr$
$wv_1(r) \leftarrow v_1 / \sqrt{m}$
while $i \leqslant n$ **do**
  $v' \leftarrow v_i$
  for j from 1 to $i-1$ **do**
    $h_j \leftarrow \int_0^1 v_i(r) wv_j(r) r dr$
    $v' \leftarrow v' - h_j wv_j$
  end for
  $m \leftarrow \int_0^1 v'(r) v'(r) r dr$
  $wv_i \leftarrow v' / \sqrt{m}$
end while