



# Locally enhancing fractal descriptors by using the non-additive entropy<sup>☆</sup>



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## ABSTRACT

This work proposes to increase the discrimination ability of fractal descriptors by using information concerning the degree of local organization and homogeneity in the curve of descriptors. Such information is quantified by the non-additive entropy computed along the neighbourhood of each point in the curve of fractal measures. These values are used as features for texture images and assessed in the classification of a well-known data set, to know, Brodatz, as well as in an application to the automatic identification of plant species based on images of leaves. The results are compared using different approaches (combining different lengths of windows and different entropy parameters) as well as with other state-of-the-art and well-known texture descriptors in the literature. The achieved results outperformed the other approaches, suggesting the proposal as being an interesting strategy to provide even more precise and rich features for image analysis.

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## 1. Introduction

A texture image (also named visual texture) is a grey-level image presenting particular patterns of pixel distributions along different scales, being all these distributions important to characterize the object represented in that image. Texture analysis plays a central role in many problems of image analysis where the information is enclosed along the local/spectral distribution of pixel intensities rather than only on a binary silhouette. Such research has been applied to a number of practical areas [1,20].

This is a challenging task once texture images use to be much more complex than binary objects. Furthermore, in many real situations like the supposed here, there is no information from colour, which can facilitate the analysis when the colour acquisition is possible.

In this context, a number of approaches have been proposed during the last decades, using many different types of strategies with the aim of extracting the most meaningful information [9,12,13,15]. Among these methods, fractal geometry has gathered significant attention from the literature on analysis of images in the last years [2,6,7,21]. Such interest is justified mainly by the essence of Fractal Geometry, which was developed to analyse structures with particularly complex details under different scales.

In fact, a multi-scale complex behaviour can be easily found in the nature and the texture images representing such real-world objects

express this characteristic in the multiple relationships among the pixels and regions of the image. Such theoretically expected effectiveness is confirmed by the remarkable results achieved by the fractal-based features in several tasks of image analysis either on benchmark databases or on practical problems [2,8].

Among the several techniques developed to compute image descriptors using concepts from Fractal Geometry, the fractal descriptors [2,6,7], and particularly those obtained from the Bouligand–Minkowski dimension [2], have shown to be the most effective, especially for the analysis of natural images.

Despite its success and different proposed ways of obtaining fractal descriptors from the image, the relation between the descriptors has not been well explored in the literature. Actually, the Bouligand–Minkowski features are provided by a curve in bi-logarithmic scale and in this way such descriptors can be managed as a sequence of numbers with an intrinsic order. This property makes possible a neighbourhood analysis over this curve highlighting local characteristics which can be even more important than the curve itself.

Based on these assumptions, this work proposes a local-based analysis of the curve of fractal descriptors using the non-additive Tsallis entropy [19]. Like other definitions of entropy in the context of Information Theory, this is a measure of the amount of information contained in a signal, but now including a parameter  $q$  that makes more flexible the modelling of a system not so “well-behaved” as those measured by the classical Boltzmann–Gibbs–Shannon (BGS) entropy. The parametric approach also allows to analyse the data under different perspectives and possibly to test some combinations of parameters such as done in this work.

The proposed descriptors are assessed in a classical problem of image analysis, i.e., the classification of texture databases. Here, the

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tests were performed on two databases: the well-known Brodatz [3] textures and a database of images collected from leaves of Brazilian plants for the purpose of identifying the respective species [4]. Finally, the precision in the classification is compared to the original fractal descriptors as well as to other extensively used methods in the literature. The enhanced descriptors outperform all of them, demonstrating its high potential to be a worthy tool for the analysis of textures in a wide range of related problems, mainly in those cases where the image variability is subtle and not restrict to specific scales.

## 2. Fractal descriptors

Although there are different fractal descriptors proposed in the literature [2,6,7], here the focus is on the Bouligand–Minkowski descriptors, which has previously demonstrated its efficiency in texture analysis [2] and is proposed to be enhanced in this work.

The method starts by mapping the grey-level image into a tree-dimensional cloud of points  $\mathcal{C}$ . Thus given an image  $I: [1, M] \times [1, N] \rightarrow \mathbb{N}$  the cloud is given by a set of points with coordinates  $(x, y, z)$ , such that:

$$\mathcal{C} = \{(x, y, z) | I(x, y) = z\}. \quad (1)$$

In the following, each point is dilated by a sphere with radius  $r$ . As these points are simultaneously dilated they start to collide among themselves resulting in a dilated mass  $\mathcal{C}_r$  of points in the space. The coordinate  $(x_r, y_r, z_r)$  of each point in this dilated structure satisfies:

$$\sqrt{(x_r - x)^2 + (y_r - y)^2 + (z_r - z)^2} \leq r, \forall (x, y, z) \in \mathcal{C}. \quad (2)$$

Finally, the Bouligand–Minkowski descriptors are obtained from the volumes  $V(r)$  of  $\mathcal{C}_r$  when  $r$  is varied between 0 and  $r_{max}$ . More details on the computation of  $\mathcal{C}_r$  and how the descriptors are obtained can be found in [2].

## 3. Non-additive entropy

The non-additive entropy [19]  $S_q$  is an alternative to the classical BGS entropy to make the concept more flexible in the modelling of generic physical systems. Basically, it is a generalization of BGS entropy and defined for a distribution  $p(x)$  through:

$$S_q(x) = \sum p(x) \ln_q(1/p(x)), \quad (3)$$

where  $\ln_q$  ( $q$ -logarithm) is given by:  $\ln_q(x) = (x^{1-q} - 1)/(1 - q)$ . The entropy parameter  $q$  is real and chosen to better describe each particular system. When  $q \rightarrow 1$ , the non-additive entropy is defined in the same way as its BGS version:

$$S_1 = \sum p(x) \ln(p(x)), \quad (4)$$

In the analysis of signals and images, BGS (Boltzmann–Gibbs–Shannon) entropy [17] is a well-known metric to quantify the predictability of a sequence of measures acquired in the time, for example. The non-additive entropy includes the parameter  $q$  in this analysis, now making possible to quantify the non-additivity of the model. This property is particularly useful in the analysis of images from the nature, where multifractal structures often use to arise [11,14,16,18].

## 4. Proposed method

This work proposes to use the non-additive entropy to extract local information from the Bouligand–Minkowski descriptors. Such information is employed to obtain more powerful texture descriptors, taking into account that the set of values of  $V(r)$  is naturally ordered (by the dilation radius) and thus they can be easily managed as a discrete signal (curve) presenting rich local variation.

Actually, using the well-known work of Claude Shannon in Information Theory [17], BGS entropy has been extensively used as a statistical representation of discrete signals carrying information. Here,

the fractal curve  $\log(V(r))$  can be interpreted as a discrete signal without any loss of generality and the carried information is the dilation volume at each scale. Understanding how such values are organised (entropically speaking) gives us useful information regarding the degree of homogeneity of the multiscale fractal dimension and, as an indirect consequence, of the complexity of the texture image (at the respective scales).

A problem with using BGS, however, is the assumption that the different states of a system (or the scales in the image in this case) are linearly related (additivity). When modelling systems as complex and unpredictable as a texture image, this assumption imposes a severe limitation. On the other hand, Tsallis entropy can overcome such obstacles by including the non-additivity parameter ( $q$ ). Parameter  $q$  also allows the statistical representation of long-range interaction usually present in complex systems and especially important in a multifractal context, also assumed for textures in the original theory of fractal descriptors.

The proposal can be essentially divided into three steps. At first, the curve of fractal descriptors (obtained from  $\log(V(r))$ ) is split into overlapping windows with length  $r$  centred at each point of the original signal. In the following, the non-additive entropy is computed over the piece of the curve within each window.

In this way, if the set of  $n$  Bouligand–Minkowski descriptors is represented in a functional notation as  $D(k)$ , being  $1 \leq k \leq n$ , then the operator  $\hat{K}$  acts on  $D$  in the following way:

$$\hat{K}(i) = \cup_{k=i}^n S_q(\cup_{k=i}^{i+r} (D(k))), \quad (5)$$

where  $\cup$  here denotes the concatenation of numbers respecting the original order. Fig. 1 illustrates the steps involved to compute the local non-additive entropy. At the top, the original curve of Bouligand–Minkowski descriptors and two points identified (1 and 2). In the following row, the regions of points 1 and 2 are zoomed in, showing the points in the window used to compute the entropy (equation just below). Finally, at the bottom, the curve of entropies, highlighting the respective points 1 and 2.

Finally, the entropy measures are put together and submitted to a Principal Component Analysis (PCA) [5] to reduce the dimensionality. PCA is also applied to combine different parameters in the method. The PCA scores are in fact the proposed descriptors and can posteriorly be used in any task of analysis.

## 5. Experiments

The performance of the proposed method is evaluated over a widely used database of texture images named Brodatz [3]. This is a set of grey-level pictures from the real-world and scanned from a book of architecture. Here, each one of 111 images were divided into 16 non-overlapping windows, each window with a dimension of  $128 \times 128$ . Fig. 2(a) shows some samples of images from this database.

The method is also applied to a practical problem, to know, the automatic identification of species (taxonomy) of plants from the Brazilian flora. For this purpose a database named 1200tex [4] was employed, comprising 20 classes (species) with 20 samples per species. From each sample, three  $128 \times 128$  images were collected by a desktop scanner, after removing impurities and aligning the central axis of the leaf. Fig. 2(b) exemplifies some samples of images from the database.

The image descriptors are computed from each sample and such features are classified by a Linear Discriminant Analysis (LDA) classifier [5]. Besides the original fractal descriptors (without entropy), the proposed method is compared to other classical and state-of-the-art approaches of image analysis, to know, Grey-Level Co-occurrence Matrix (GLCM) [9], Gabor-wavelets [12] and Local Binary Patterns [15]. Ideal parameter settings employed in the respective works are used. Finally, the results are compared in terms of global and per-class success rates and number of descriptors.

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