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# Fast and reliable noise level estimation based on local statistic☆

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ABSTRACT

Noise level is an important premise of many image processing applications. This letter presents an automatic noise estimation method based on local statistic for additive white Gaussian noise (WGN). Analysis of the distribution of local variance shows that when local variances are not greater than the threshold that satisfies a special condition, their average is always linearly correlated with the real noise variance. Thus the actual noise variance can be obtained from these patches. Based on this idea, this letter provides an iterative process to select flat blocks, and estimates noise variance from these homogeneous patches using principal components analysis. Addressing challenges in noise estimation has major contributions to (1) studies on the distribution of local statistic and (2) an iterative process for choosing flat patches, which is the fundamental work of patch-based methods. The experiment results show that the proposed algorithm works well over a large range of visual content and noise conditions, and performs well in multiplicative noise. Compared with several conventional noise estimators, it yields best performance and faster running speed.



## 1. Introduction

Digital images can be easily distorted by noise during capturing, processing and transmission. Many image and video processing algorithms, such as de-noising [6,10], compression [31], segmentation [24] and image quality assessment [14,32], need to be corrected according the noise variance, but it is not always available in practice. Thus, accurate and reliable blind noise estimation becomes an important research topic.

The key of noise estimation for a single image is how to prepare an ideal dataset for noise estimation. Based on how to choose the dataset, the existing noise estimation methods can be mainly classified into three categories: transform-based methods, filter-based methods and patch-based methods.

In the transform-based methods, the input image is first transformed into domains which are then used for noise estimation. The methods proposed in [7,15] use the wavelet transform to isolate the noise information in the first diagonal band coefficients, and then estimate noise level from the absolute value of these coefficients, but these methods tend to overestimate in high frequency images (images with strong structures). The algorithms in [18,19] estimate noise variance in singular value decomposition domain. These methods divide the singular values into two parts and assume that the part contributed by noise-free image is fixed. However, this part maybe changes with noise variance, especially for those images with low noise level or high frequency. As a result, these methods tend to overestimate the noise when noise variance is small or image includes abundant features. Ghazi and Erdogan [9] estimate noise level via matching moments of coefficients in discrete cosine transform (DCT) or discrete wavelet transform (DWT) domains, but DCT and DWT are difficult to completely separate image signal from the observed image.

The filter-based algorithms filter the image with a low-pass filter at first and then estimate the noise variance using the difference between the noisy image and the filtered image [17,25,28]. The main challenge is that the difference between the images is assumed to be the pure noise but this assumption is not held in general, because a low-pass filtered image is not the ground truth image, especially for an image with strong structures. Therefore, most filter-based approaches perform well for image with less texture, but tend to overestimate the noise in high frequency images [12,23].

In the patch-based approaches, the noisy image is tessellated into many patches and the noise variance is estimated using a set of specifically chosen homogeneous patches [11,29]. Different patch-based methods are different in how to choose flat patches. Ajá-Fernandez et al. [2] estimates noise from the mode of local

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statistics with premise that most image patches are flat, which does not always hold in practice. In methods [20,26], the noise level is estimated as the smallest eigenvalue of the homogeneous patch covariance matrix. However, the minimal eigenvalue is usually smaller than the true noise variance if there are a few chosen flat blocks. Chen et al. [5] construct the covariance matrix by using all image blocks in three color channels, so that it maybe fails in some gray-level images. Ant colony optimization technique is used to select uniform blocks in [30], nevertheless, it consumes huge computational complexity. Amer and Dubois first select the most homogeneous patches by using a Laplacian operator, then take the blocks with local variances close to the selected block as uniform areas [3]. This method is accurate for high frequency image, but it has low reliability and tends to underestimate. The improved approaches are also based on the most uniform block with very small local variance, hence, they cannot absolutely avoid underestimation [8,13] .

If images are degraded by additive Gaussian noise, the analysis for the probability density function (*pdf*) of local variance shows that there must exist a unique set of homogeneous patches with smaller local variances and the average of these local variances is always linearly correlated with the real noise variance. That is to say, we can get the actual noise level from these homogeneous patches. Based on this idea, this letter presents a novel noise estimation method which does not underestimate for low structure images, but also works well for high frequency images.

The rest of this letter is arranged as follows: Section 2 analyses the local statistic and describes the proposed noise estimation method. Section 3 simulates the proposed method and compares the proposed method with some classical approaches. Section 4 concludes this letter.

## 2. Noise estimation algorithm

In this section, the local variance of patch is analyzed, an iterative process to extract flat blocks is provided, and the noise level is evaluated by using principal components analysis.

### 2.1. Local variance

If the noise-free image $s$ is distorted by a zero-mean additive WGN $\eta$ with variance $\sigma^2$ , the noisy image $v$ is

$$v = s + \eta \tag{1}$$

Suppose that $P_W = \{v_{ij}; i = 1, \dots, W, j = 1, \dots, W\}$ is a $W \times W$ sample block over the noisy image, the local variance is

$$\sigma_P^2 = \sum_{v_{ij} \in P_W} (v_{ij} - \mu)^2 / (W^2 - 1) \tag{2}$$

where $\mu$ is the average intensity of pixels in patch $P_W$.

Because homogeneous patch contains only pure noise, there is $(W^2 - 1)\sigma_P^2/\sigma^2 \sim \chi^2(r)(r = W^2 - 1)$ for a flat patch, where $\chi^2(r)$ is a chi-square with $r$ degrees of freedom [21]. Therefore, the distribution of $\sigma_P^2$ is Gamma distribution with the shape parameter $r/2$ and the scale parameter $2\sigma^2/r$ , *i.e.*$\sigma_P^2 \sim \gamma(r/2, 2\sigma^2/r)$, whose *pdf* is

$$f(x) = \begin{cases} \dfrac{1}{\Gamma(r/2)} x^{\frac{r}{2}-1} \left(\dfrac{r}{2\sigma^2}\right)^{\frac{r}{2}} e^{-\frac{r}{2\sigma^2}x}, & x \geq 0 \\ 0, & otherwise \end{cases} \tag{3}$$

where $\Gamma(\cdot)$ denotes the Gamma function. The mean of $\sigma_P^2$ turns out to be the variance of noise, *i.e.*,

$$E(\sigma_P^2) = \frac{r}{2} \times \frac{2\sigma^2}{r} = \sigma^2 \tag{4}$$

**Table 1**
Different $\lambda$ and $\rho$ according to $\delta$.

| $\delta$ | $W = 5$ | | $W = 7$ | | $W = 9$ | |
|---|---|---|---|---|---|---|
| | $\lambda$ | $\rho$ | $\lambda$ | $\rho$ | $\lambda$ | $\rho$ |
| 0.7 | 1.3267 | 0.8510 | 1.2240 | 0.8956 | 1.1707 | 0.9196 |
| 0.8 | 1.3807 | 0.8918 | 1.2613 | 0.9248 | 1.1992 | 0.9424 |
| 0.9 | 1.4758 | 0.9327 | 1.3261 | 0.9569 | 1.2482 | 0.9672 |

where $E(\cdot)$ denotes the expectation. This is the reason why we usually estimate the noise by averaging the local variances.

For homogeneous blocks, we can get the following theorem

**Theorem 1.** *There must exist a unique* $\tilde{\sigma}^2(\tilde{\sigma} > 0)$ *such that*

$$\tilde{\sigma}^2 = E(\sigma_P^2 \,|\, \sigma_P^2 \leq \lambda\tilde{\sigma}^2) \tag{5}$$

$$\tilde{\sigma}^2 = \rho\sigma^2 \tag{6}$$

*with*

$$\rho = \frac{\int_0^{F^{-1}(\delta, r/2, 2/r)} xz(x)dx}{\int_0^{F^{-1}(\delta, r/2, 2/r)} z(x)dx} \tag{7}$$

$$\lambda = F^{-1}(\delta, r/2, 2/r)/\rho \tag{8}$$

*where* $z(x) = \sigma^2 f(\sigma^2 x)$ *represents the pdf of* $\gamma(r/2, 2/r)$, $F^{-1}(\cdot)$ *is the inverse Gamma cumulative distribution function with* $r/2$ *shape parameter and* $2/r$ *scale parameter, and* $\delta$, *which is a given significance level, denotes the probability of that local variance is no more than* $\lambda\tilde{\sigma}^2$, *i.e.*, $P(\sigma_P^2 \leq \lambda\tilde{\sigma}^2) = \delta$ ($P(\cdot)$ *represents the probability*).

**Proof.** Since $\delta$ is known, *i.e.*,

$$P(\sigma_P^2 \leq \lambda\tilde{\sigma}^2) = \delta \tag{9}$$

we have

$$P(\sigma_P^2/\sigma^2 \leq \lambda\tilde{\sigma}^2/\sigma^2) = \delta \tag{10}$$

It is easy to know $\sigma_P^2/\sigma^2 \sim \gamma(r/2, 2/r)$, then

$$\lambda\tilde{\sigma}^2/\sigma^2 = F^{-1}(\delta, r/2, 2/r) \tag{11}$$

For the similar reason, one can obtain

$$\begin{aligned} E(\sigma_P^2 \,|\, \sigma_P^2 \leq \lambda\tilde{\sigma}^2) &= \sigma^2 E(\sigma_P^2/\sigma^2 \,|\, \sigma_P^2/\sigma^2 \leq \lambda\tilde{\sigma}^2/\sigma^2) \\ &= \sigma^2 \frac{\int_0^{F^{-1}(\delta, r/2, 2/r)} xz(x)dx}{\int_0^{F^{-1}(\delta, r/2, 2/r)} z(x)dx} \end{aligned} \tag{12}$$

Defining $\rho = \int_0^{F^{-1}(\delta, r/2, 2/r)} xz(x)dx / \int_0^{F^{-1}(\delta, r/2, 2/r)} z(x)dx$, Eq. (12) is

$$E(\sigma_P^2 \,|\, \sigma_P^2 \leq \lambda\tilde{\sigma}^2) = \rho\sigma^2 \tag{13}$$

Based on Eqs. (11) and (13), there is

$$\begin{aligned} \tilde{\sigma}^2 &= F^{-1}(\delta, r/2, 2/r)\sigma^2/\lambda \\ &= (F^{-1}(\delta, r/2, 2/r)/\rho)/\lambda E\left(\sigma_P^2 \,|\, \sigma_P^2 \leq \lambda\tilde{\sigma}^2\right) \end{aligned} \tag{14}$$

Supposing $\lambda = F^{-1}(\delta, r/2, 2/r)/\rho$, the following is obtained from Eqs. (13) and (14)

$$\tilde{\sigma}^2 = E(\sigma_P^2 \,|\, \sigma_P^2 \leq \lambda\tilde{\sigma}^2) = \rho\sigma^2 \tag{15}$$

Since $\delta$ is a given constant, $\rho$ and $\lambda$ are constants also. Thus, $\tilde{\sigma}^2$ is unique and linearly correlated with $\sigma^2$. With different patch size, the $\lambda$ and $\rho$ according to the $\delta$ are shown in Table 1. □

Theorem 1 means that we can get the actual noise variance from all the homogeneous blocks with local variances no more than $\lambda\tilde{\sigma}^2$, where $\tilde{\sigma}^2$ is the average of their local variances. It can work well for high structure image because many non-homogeneous patches with greater than $\lambda\tilde{\sigma}^2$ local variances are rejected.