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Optical nonlinearities associated to applied electric fields in parabolic two-dimensional quantum rings

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ABSTRACT

The linear and nonlinear optical absorption as well as the linear and nonlinear corrections to the refractive index are calculated in a disc shaped quantum dot under the effect of an external magnetic field and parabolic and inverse square confining potentials. The exact solutions for the two-dimensional motion of the conduction band electrons are used as the basis for a perturbation-theory treatment of the effect of a static applied electric field. In general terms, the variation of one of the different potential energy parameters – for a fixed configuration of the remaining ones – leads to either blueshifts or redshifts of the resonant peaks as well as to distinct rates of change for their amplitudes.

1. Introduction

Parabolically shaped confining potential profiles have appeared in many works dealing with the description of electronic and optical properties of low-dimensional semiconducting heterostructures [1–9]. Among these systems, the quantum dots (QDs) and quantum rings (QRs) are subjects of great interest due to their prospective applications to the design and fabrication of optoelectronic devices [10–13]. Going beyond the case of single nanostructures, it has been seen that the artificial molecules consisting of coupled QDs or QRs are particularly promising for quantum information processing [14] and terahertz devices [15]. It is also possible to mention a number of works devoted to the study of linear and nonlinear optical properties in parabolic QDs [16,17]. Furthermore, a particular type of quasi-zero-dimensional system known as quantum disc (QDC) – or disc-shaped quantum dot – has also attracted some attention [18–21]. Precisely, in this work, we are going to focus our attention on this latter type of structures.

Incorporating an externally applied electric fields may result in significant modifications of the electron energy spectrum in quantum heterostructures. This certainly affects a large number of properties in them. Compilation of the large amount of scientific publications on that matter is beyond the scope of this paper; but we can refer here to several previous works that have considered the effects of electric fields on parabolically confined nanosystems [5,22,23]. One of the most important consequences of the application of a static electric field is the possibility of obtaining rather strong nonlinear optical responses, resulting from the consequent increment in the expected values of the electron dipole moment, as it was originally shown by Ahn and Chuang in the case of quantum wells [24]. In addition, optical nonlinearities in QDs with parabolic potentials have been recently reported [21,25–28].

The present paper is devoted to the study of the influence of the application of a *x*-directed in-plane static electric field on the electronic states in two-dimensional quantum discs under the combination of two distinct confining profiles: a parabolic-type, and the one with inverse square dependence. All this is complemented with the presence of an externally applied magnetic field.

On the other hand, the inverse square potential function has previously appeared as a model for the inter-particle interaction in a study of the quantum problem of N particles in a two-dimensional parabolic potential under the effect of a magnetic field [29]. We have already put forward a discussion about the possibility of that, using this inverse square potential, together with the other two potential field influences, might lead to a rather direct modeling for the states of carriers confined in a two-dimensional semiconducting QR, where the single electron eigenstates of the corresponding Schrödinger-like conduction band effective mass equation can be exactly described via analytical expressions [21]. (These authors have become aware that the paper of Ref. [27], containing the same mathematical solution, was in the process of revision at the time we submitted ours, and appeared published earlier.) With the use of the obtained energy

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levels and wavefunctions, we will apply the perturbation theory to obtain the corrections to the energies and wavefunctions associated with the electric field potential. We carefully discuss the conditions of applicability of the perturbation theory, based on the strength of the static applied electric field. It will be shown that they vary in regard with the particular set of values that characterize the total confining potential. Then, with the used of the perturbed electron states, it will be possible to calculate the linear and nonlinear optical absorption and relative refractive index variation in a GaAs-based system with such a geometry and external fields. The paper is organized as follows. In Section 2 we describe the theoretical framework. Section 3 is dedicated to the discussion of the obtained results, and our conclusions are given in Section 4.

2. Theoretical framework

In this work we are considering the motion of conduction band electrons in the QR configuration arising from the combination of a two-dimensional disc-shaped quantum dot (DSQD) with the effect of four distinct sources of potential energy: A parabolic confining one, an inverse square potential energy function, a static magnetic field **B**, oriented along the positive z-direction – normal to the plane – and a x-oriented DC-electric field [in Cartesian coordinates $(-F,0,0)$], taken as a perturbation interaction. Here, the polar system (r,φ) is a suitable set of coordinates for the description of the allowed quantum states. The Hamiltonian of the system, within the framework of the effective mass approximation, is given by

$$H = H_0 - eFr\cos\varphi, \quad (1)$$

with H_0 being the unperturbed Hamiltonian;

$$H_0 = \frac{1}{2m^*}\left[\mathbf{p} + \frac{e}{c}\mathbf{A}\right]^2 + V(\mathbf{r}), \quad (2)$$

where e, m^* and c are the absolute value of the electron charge, the electron effective mass, and speed of light, respectively. $\mathbf{A} = (A_r = 0, A_\varphi = Br/2, A_z = 0)$ is the vector potential of the static magnetic field. The confining potential, $V(\mathbf{r})$, combines the parabolic and inverse square potential functions;

$$V(\mathbf{r}) = \frac{1}{2}m^*\omega_0^2 r^2 + \frac{\hbar^2}{2m^*}\frac{\lambda^2}{r^2}, \quad (3)$$

where ω_0 represents the confinement frequency of the parabolic potential and λ – a dimensionless parameter – characterizes the strength of the inverse square potential.

The problem with $F \equiv 0$ has analytical solutions. Using the Coulomb gauge, the Schrödinger equation given by the Hamiltonian in Eq. (2), takes the form (in polar coordinates),

$$\left[-\frac{\hbar^2}{2m^*}\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}\right) + \frac{1}{2}m^*\Omega^2 r^2 + \frac{\hbar^2}{2m^*}\frac{\lambda^2}{r^2} + \frac{\omega_c}{2}L_z\right]\psi^{(0)} = E^{(0)}\psi^{(0)}, \quad (4)$$

where $\omega_c = eB/m^*c$ is known as the cyclotron frequency, $\Omega = \sqrt{\omega_0^2 + \omega_c^2/4}$ is the total confinement frequency in the magnetic field, $E^{(0)}$ is the energy eigenvalue of the system without taking into account the electric field interaction (Eq. (4)), and L_z is the orbital angular momentum operator along the z-direction. To find the solution of Eq. (4) that corresponds to a two-dimensional eigenstate $\psi^{(0)}$, it is customary to propose

$$\psi(r,\varphi)^{(0)}_{mn} = N_{mn}\frac{\chi(r)}{\sqrt{r}}e^{im\varphi}, \quad (5)$$

where N_{mn}, is a normalization constant and $m = 0, \pm 1, \pm 2 \ldots$ is the magnetic quantum number (the angular part of function (5) is an eigenstate of $\hat{L}_z$ with eigenvalue $m\hbar$). The radial function $\chi(r)$ is given by [21,27]

$$\chi(r) = r^{t_m} e^{-r^2/2\eta^2} F(-n, t_m + 1/2, r^2/\eta^2), \quad (6)$$

where $\eta = \sqrt{\hbar/m^*\Omega}$, $t_m = 1/2 + \sqrt{\lambda^2 + m^2}$, and n is a non-negative integer. The function $F(a,b,x)$ is the Kummer confluent hypergeometric function which, in the case of $a = -n$ leads to an algebraic expression of n-th order in both x and b, related with the associated Laguerre polynomial. The formula that gives the allowed unperturbed energies is,

$$E^{(0)}_{m,n} = (2n + 1 + \sqrt{\lambda^2 + m^2})\hbar\sqrt{\omega_0^2 + \frac{\omega_c^2}{4}} + \frac{m\hbar\omega_c}{2}. \quad (7)$$

As can be seen from Eq. (7), the presence of a magnetic field (non-zero ω_c) lifts the degeneracy in the energy spectrum appearing in the zero-field case that associates with the quantum number m; and the systems's ground state corresponds to the quantity $E^{(0)}_{0,0}$. The calculation of the wavefunctions and energy levels when there is a non-zero applied electric field goes through the use of the perturbation theory in the non-degenerate case (first-order corrections to the wavefunctions and second-order corrections to the energies). The criterion for the applicability of the perturbative approach established here demands the obtention of energy corrections not above the 10% of the unperturbed levels. The fulfillment of this condition will depend on the particular configuration of parabolic and inverse square potential strengths and magnetic and electric field intensities. For instance, for fixed values of ω_0 and λ, the consideration of the applied DC electric field as a perturbation is related with the value of B. The greater the magnetic field intensity, the larger the range of values of F that induces corrections below the 10%.

Here, we are interested in the effect of the electric field on the non-linear optical properties in the system. In particular, we shall investigate inter-level-related light absorption as well as the corrections to the refractive index. Assuming the polarization of the incident radiation along the in-plane x-axis, we may write the electric dipole transition matrix elements as

$$M_{ij} = \langle\psi_i^{(1)}|r\cos\varphi|\psi_j^{(1)}\rangle, \quad (8)$$

where $\psi_i^{(1)}$ $(i = 0,\ 1)$ are the corresponding first-order-corrected state wavefunctions.

The linear and third-order nonlinear optical absorption coefficients are given respectively by

$$\alpha^{(1)}(\omega) = \omega\sqrt{\frac{\mu}{\epsilon_R}}\frac{|M_{01}|^2 e^2\sigma_v\hbar\Gamma_0}{(E_{01} - \hbar\omega)^2 + (\hbar\Gamma_0)^2} \quad (9)$$

and

$$\alpha^{(3)}(\omega, I) = -\omega\sqrt{\frac{\mu}{\epsilon_R}}\left(\frac{I}{2\epsilon_0 nc}\right)\frac{e^4|M_{01}|^2\sigma_v\hbar\Gamma_0}{[(E_{01} - \hbar\omega)^2 + (\hbar\Gamma_0)^2]^2} \times\left[4|M_{01}|^2 - \frac{\delta M[3E_{01}^2 - 4\hbar\omega E_{01} + \hbar^2(\omega^2 - \Gamma_0^2)]}{E_{01}^2 + (\hbar\Gamma_0)^2}\right] \quad (10)$$

where $\delta M = (M_{11} - M_{00})^2$, $E_{01} = E_1 - E_0$, σ_v is the electron density of the QR, μ is the permeability of the system, $\epsilon_R = \epsilon_0\epsilon$ (ϵ_0 is the vacuum permittivity and ϵ the GaAs static dielectric constant), $n = \sqrt{\epsilon}$ is the refractive index, $\hbar\omega$ is the incident photon energy, and $I = 2\epsilon_0 n_r c|\tilde{E}|^2$ is the incident optical intensity. Therefore, the total optical absorption coefficients can be written as

$$\alpha(\omega, I) = \alpha^{(1)}(\omega) + \alpha^{(3)}(\omega, I). \quad (11)$$

In a similar manner it is possible to obtain the linear and third-order nonlinear relative refractive index change whose expressions