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Impact of variable thermal conductivity in doubly stratified chemically reactive flow subject to non-Fourier heat flux theory

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ABSTRACT

This attempt reports the influence of variable thermal conductivity over an impermeable surface stretching in a nonlinear manner. Flow formulation is developed considering stagnation point and rheological expressions of second grade liquid. Different from the traditional literature, a new concept of heat flux covering paradox of heat conduction is imposed. Such concept has been used in view of Cattaneo-Christov theory. Besides this first order chemical reaction and double stratification are also considered. The subjected problems are modeled first and then non-dimensionalized. Computations for highly nonlinear problems are presented. The derived expressions are acceptable for convergence. Velocity, temperature, concentration, skin friction and Sherwood number are described through graphs for meaningful discussion considering important variables. Our computed analysis reveals that impacts of local second grade parameter and ratio of velocities have similar behavior on velocity distribution. Moreover the consideration of variable thermal conductivity improves the temperature and associated thermal boundary layer thickness.

1. Introduction

In perspective of developing requirement for chemical processes in hydrometallurgical and chemical industries demands the heat and mass transfer analysis is quite significant. No doubt existence of external mass in a liquid mass creates chemical reaction which can exhibit either independently from anyone else or as combinations with a liquid. In numerous chemical engineering processes, a chemical reaction arises between external mass and the liquid in which the surface is moving. Several industrial procedures involve these reactions, for instance, food processing, manufacturing of glassware or ceramics and polymer production. There are two types of reactions i.e. heterogeneous and homogeneous processes. In first order chemical reaction the reaction rate is directly proportional to the concentration. Several investigators consider these processes under different physical aspects. For instance, Anjalidevi and Kandasamy [1] studied the characteristics of chemical reaction towards a horizontal plate. Chemical reaction aspects in nonlinear stretchable flow of second grade liquid subject to magnetohydrodynamics are addressed by Cortell [2]. Flow analysis in shock tube with chemical reaction and boundary layer interaction is addressed by Chen and Sun [3]. Mythili and Sivaraj [4] studied the chemically reactive flow of Casson liquid towards a vertical cone. Chemical reaction and radiation effects in flow of Jeffrey liquid by stretched surface is studied by Narayana and Babu [5]. Sankad and Dhange [6] studied the impacts of peristaltic pumping and chemical reactions in viscous material filling porous space. Hayat et al. [7] addressed impact of non-Fourier heat flux in stagnation point flow of Maxwell liquid considering homogeneous/heterogeneous processes. Analysis of MHD viscoelastic liquid towards vertical stretched surface with heat generation/absorption and chemical reactions is scrutinized by Jena et al. [8]. Khan and Hashim [9] developed numeric solutions for chemically reacting Carreau liquid considering variable thermal conductivity and multiple slip effects. Impacts of homogeneous/heterogeneous processes in nonlinear stretchable flow of viscous, Carreau and Casson liquids are disclosed by Hayat et al. [10,11] and Khan et al. [12].

Heat transfer through Cattaneo-Christov theory [13,14] has gained special consideration from the research community. Such consideration is due to the fact that law of heat conduction provided by Fourier [15] has limitation for development of parabolic energy expression. The energy expression clearly illustrates that the initial

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disturbance is immediately observed by the medium. This argument is named as "Paradox of heat conduction". Fourier's law of heat conduction is modified in various ways and conditions to avoid this feature. Haddad [16] analyzed the thermal instability of Cattaneo-Christov heat flux in Brinkman porous material problems. Uniqueness of expressions for energy equation through Cattaneo-Christov theory is reported by Ciarletta and Straughan [17]. Straughan [18] analyzed the model of heat transfer reported by [13,14] in viscous material for thermal convection. Models of Gene-culture shock and acoustic waves in view of Cattaneo-Christov theory are presented by Straughan [19,20]. Effectiveness of non-Fourier heat flux model in three-dimensional (3D) stretched flow of Burgers liquid is studied by Khan and Khan [21]. Khan et al. [22] considered chemically reactive stretched flow of Burgers liquid model subject to non-Fourier heat flux. They constructed HAM solutions. Analysis of Jeffrey liquid in the presence of non-Fourier heat flux model and homogeneous-heterogeneous reactions is addressed by Hayat et al. [23]. Characteristics of time dependent thermal conductivity in flow of generalized Burgers fluid is analyzed by Waqas et al. [24]. They studied heat transfer through non-Fourier heat flux theory. Khan et al. [25] addressed temperature-dependent thermal conductivity aspects in Sisko fluid flow considering non-Fourier double diffusion concept. Hashim and Khan [26] implemented non-Fourier flux concept to investigate the stagnation point flow of Carreau liquid in the presence of heterogeneous/homogeneous processes. Analytic solutions for stretchable flow of Eyring-Powell liquid in the presence of temperature-dependent conductivity and non-Fourier double diffusion aspects are developed by Waqas et al. [27].

Stratification of liquid in formation/deposition of layers arises due to presence of different liquids or differences in temperature and concentration. The impact of stratification is essential in numerous energy storing systems and heat rejection procedure. Several studies under different physical aspects have been reported in this direction. For instance Bansod and Jadhav [28] analyzed stratified mixed convection flow filling porous space towards vertical surface. Characteristics of double stratification in mixed convection flow of viscous nanoliquid saturated porous medium is presented by Srinivasacharya and Surender [29]. Ibrahim and Makinde [30] scrutinized the effectiveness of double stratification in stretched flow of nanoliquid by a vertical surface. Combined effects of MHD, mixed convection and double stratification on radiative stretched flow of Maxwell nanoliquid is reported by Hussain et al. [31]. Hayat et al. [32] modeled and analyzed the doubly stratified flow of thixotropic nanoliquid with heat generation/absorption and mixed convection.

The construction of sheeting materials which incorporates both polymer and metal sheets emerges in various processes [33–40]. Boundary layer approach provided by Sakiadis [41,42] and Crane [43] is utilized by several researchers under different physical aspects. For instance Turkyilmazoglu [44] explored heat transfer features in flow of micropolar liquid by a porous shrinking surface. Characteristics of viscous dissipation and mixed convection on MHD flow towards stretched surface is studied by Bhukta et al. [45]. Pal and Mondal [46] considered flow of viscous nanoliquid over a stretched/shrinked surface for the impacts of radiation, mixed convection, viscous dissipation and heat generation in a permeable medium. Effectiveness of thermal radiation in MHD flow of nanofluid towards stretched surface with slip effects is presented by Mahanthesh et al. [47].

In perspective of the aforementioned literature survey it is noted that flows with variable thickness are less attended. Such flows have significant demands in mechanical, civil, marine and aeronautical structures. No doubt the weight of structural elements can be minimized utilizing variable thickness. Few recent studies in this direction can be mentioned through [48–52]. With this view point our attention here is to scrutinize the impacts of chemical reaction and double stratification in variable thermal conductivity flow of second grade material over nonlinear stretched surface with variable thickness. Idea of Cattaneo [13] and Christov [14] is utilized to model energy expression. Besides this homotopic algorithm [53–60] is utilized to compute the nonlinear analysis. The velocity, temperature, concentration, skin friction and Sherwood number are examined for various sundry variables through the formulation.

2. Formulation

Our goal here is to explore the chemically reactive stagnation point flow of an incompressible second grade material towards a stretching surface. Thickness of nonlinear stretching surface is variable i.e. $y = \delta(x+b)^{\frac{1-n}{2}}$. The concept provided by Cattaneo [13] and Christov [14] is imposed to discuss the salient feature of heat transfer. Besides this temperature dependent thermal conductivity and double stratifications are considered. Moreover variable temperature and concentration at and away from the sheet are denoted by (T_w, C_w) and (T_∞, C_∞) respectively. Detailed flow assumptions can be understood through Fig. 1. The governing equations for present flow with heat and mass transfers are as follows:

$$\text{div} \quad \mathbf{V} = 0, \tag{1}$$

$$\rho_p (\mathbf{V}\cdot\nabla)\mathbf{V} = \nabla\cdot\boldsymbol{\tau}, \tag{2}$$

$$(\rho c)_p (\mathbf{V}\cdot\nabla) T = -\nabla\cdot\mathbf{q}, \tag{3}$$

$$(\mathbf{V}\cdot\nabla) C = D\nabla^2 C, \tag{4}$$

$$\boldsymbol{\tau} = -p\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2, \tag{5}$$

$$\mathbf{A}_1 = (\text{grad } \mathbf{V}) + (\text{grad } \mathbf{V})^{tr}, \tag{6}$$

$$\mathbf{A}_2 = \frac{d\mathbf{A}_1}{dt} + \mathbf{A}_1\cdot\text{grad } \mathbf{V} + (\text{grad } \mathbf{V})^{tr}\cdot\mathbf{A}_1, \tag{7}$$

where $\mathbf{V}$ denotes the velocity, ρ_p the liquid density, c_p the specific heat, p the pressure, $\boldsymbol{\tau}$ the extra stress tensor, $(\mathbf{A}_1, \mathbf{A}_2)$ the (first, second) Rivlin-Ericksen tensors, (α_1, α_2) the material constants, μ the dynamic viscosity, $\mu \geq 0$, $\alpha_1 \geq 0$, $\alpha_1 + \alpha_2 = 0$, (T, C) the temperature and concentration respectively, D the molecular diffusivity of the concentration species. According to the non-Fourier concept, the following relation is satisfied for heat flux $\mathbf{q}$ [60]:

$$\mathbf{q} + \lambda[\mathbf{V}\cdot\nabla\mathbf{q} - \mathbf{q}\cdot\nabla\mathbf{V} + (\nabla\cdot\mathbf{V})\mathbf{q}] = -k(T)\nabla T, \tag{8}$$

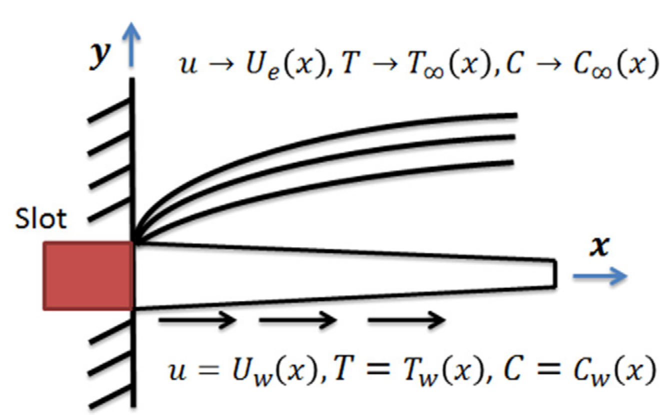

Fig. 1. Physical configuration.