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Impact of chemical reaction on third grade fluid flow with Cattaneo-Christov heat flux

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ABSTRACT

The present article deals with the two-dimensional flow of third grade fluid induced by a linear stretching sheet. Analysis of thermal relaxation time is made by using Cattaneo-Christov heat flux model. Effects of chemical reaction are also taken into account. Suitable transformations lead to a strongly nonlinear differential system which is solved through homotopic technique. Convergent series solutions are derived. Effects of the emerging parameters on the dimensionless velocity, temperature and concentration are investigated. It is found that increasing values of thermal relaxation time corresponds to low temperature. Skin friction coefficient and Sherwood number are also computed and addressed.

1. Introduction

Heat transfer phenomenon is quite significant in the industrial and biomedical applications e.g. cooling of electronic devices, nuclear reactor cooling, heat conduction in tissues, and energy production. Characteristics of heat transfer have been explored by Fourier's law of heat conduction [1] in the last two centuries. One of the major shortcomings of this model is that it produces a parabolic energy equation which means that an initial disturbance would instantly affect the system under consideration. To overcome this difficulty Cattaneo [2] amended the Fourier's law with the inclusion of thermal relaxation time in the classical Fourier's law which is defined as the time required to establish steady heat conduction once a temperature gradient is imposed. It is seen that such consideration produces hyperbolic energy equation and it allows the transportation of heat through the propagation of thermal waves with finite speed. After that Christov [3] modified the Cattaneo law by thermal relaxation time along with Oldroyd's upper-convected derivatives in order to achieve the material-invariant formulation. Straughan [4] studied Cattaneo-Christov model with thermal convection. Ciarletta and Straughan [5] proved the uniqueness of the solutions for the Cattaneo-Christov equations. Tibullo and Zampoli [6]provided the uniqueness of Cattaneo-Christov heat flux model for flow of incompressible fluids. Han et al. [7] presented stretched flow of Maxwell fluid with Cattaneo-Christov heat flux model. Mustafa [8] explored the characteristics of Cattaneo-Christov heat flux in the rotating flow of Maxwell fluid. Impact of Cattaneo-Christov heat flux in the flow over a stretching sheet with variable thickness has been studied by Hayat et al. [9]. Hayat et al. [10] also examined Cattaneo-Christov heat flux in MHD flow of Oldroyd-B fluid with homogeneous–heterogeneous reactions.

Non-Newtonian fluids have been extensively acknowledged by the investigators due to their large technological and industrial applications like paper production, polymers, coal slurries, cosmetics, oil recovery and mixture of clays etc. Non-Newtonian fluids cannot be described by single constitutive relationship. Various fluid models have been proposed to predict the salient features of non-Newtonian fluids. Rate, differential and integral types are main categories for the classification of these fluids. Third grade material is subclass of differential type fluid. It illustrates the impact of shear thickening/thinning property. Ramzan et al. [11] discussed the flow of third grade fluid with homogeneous–heterogeneous reactions. Wang et al. [12] studied MHD third grade fluid flow with heat transfer due to parallel plates. Third grade fluid flow in the presence of magnetic field is inspected by Hayat et al. [13]. Hussain et al. [14] demonstrated incompressible flow of third grade fluid past a stretching sheet with viscous dissipation. Farooq et al. [15] discussed impact of heat and mass transfer in third grade fluid flow due to stretching

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sheet. Li et al. [16] presented third grade fluid flow by rotating plates. Abbasbandy and Hayat [17] discussed third grade fluid flow due to stretching surface.

Boundary layer flows induced by a stretching surface with heat and mass transfer have gained considerable interest due to their various industrial and engineering applications including aerodynamic extrusion of plastic sheets, extrusion of polymer or rubber sheets or filament from a die, glass blowing, continuous casting, cooling of an infinite metallic plate in a cooling bath, etc. Heat transfer at the sheet has a pivotal role on the quality of final product. Extensive research has been undertaken for the stretching flows in different configurations. Ibrahim et al. [18] presented MHD stagnation point flow of nanofluid towards a stretching sheet. Malvandi et al. [19] examined slip effects on unsteady stagnation point flow of a nanofluid over a stretching sheet. Nawaz et al. [20] studied Joules and Newtonian heating effects on stagnation point flow over a stretching surface. Majeed et al. [21] explored unsteady ferromagnetic liquid flow and heat transfer analysis over a stretching sheet with the effect of dipole and prescribed heat flux. Zeeshan et al. [22] investigated effect of magnetic dipole on viscous ferro-fluid past a stretching surface with thermal radiation. Maqbool et al. [23] analyzed Hall effect on Falkner-Skan boundary layer flow of FENE-P fluid over a stretching sheet. Hayat et al. [24] examined stretched flow of Walters' B fluid with Newtonian heating. Lin et al. [25] presented MHD pseudo-plastic nanofluid unsteady flow and heat transfer in a finite thin film over stretching surface with internal heat generation.

It is noted that chemical reaction effect in the flow by moving surface is not given due attention even its various applications in bio engineering and chemical industry. Very limited attention has been paid to study the effect of chemical reaction in flow over stretching surfaces which is useful in many industrial applications processes including manufacturing of ceramics, food processing, polymer production, drying, evaporation, energy transfer in a cooling tower and the flow in a desert cooler. Hayat et al. [26] analyzed the unsteady flow with heat and mass transfer of a third grade fluid over a stretching surface in the presence of chemical reaction. Matin and Pop [27] studied the forced convection heat and mass transfer effects in flow of nanofluid through a porous channel with first order chemical reaction. The effects of chemical reaction and magnetic field in flow of couple stress fluid over a non-linearly stretching sheet is examined by Khan et al. [28]. Heat and mass transfer in MHD flow of nanofluid with chemical reaction effects have been studied by Srikanth et al. [29]. Mukhopadhyay [30] studied effects of partial slip on chemically reactive solute distribution in MHD boundary layer stagnation point flow past a stretching permeable sheet.

There are many methods to solve the nonlinear problems. Homotopy analysis method (HAM) is firstly developed by Liao in 1992 [31]. He further modified [32] with a non-zero auxiliary parameter which is also known as convergence control parameter. This parameter is a non-physical variable that provides a simple way to verify and ensure convergence of solution series. HAM always helps no matter whether there exist small/large physical parameters or not in the problem statement. It provides a convenient way to guarantee the convergence of approximation series. It also provides great freedom to choose the equation type of linear sub-problems and the base functions of solutions. The capability of the HAM to naturally show convergence of the series solution is unusual in analytical and semi-analytic approaches to nonlinear partial differential equations. Motivated by such facts, the purpose of this article is to study heat and mass transfer in the flow of third grade fluid over a stretching sheet with Cattaneo-Christov heat flux. Influence of chemical reaction is also examined. Convergent solutions are obtained by homotopy analysis method [33–38]. The behaviors of different parameters on the physical quantities of interest have been examined graphically.

2. Model development

Consider the steady two-dimensional flow of an incompressible third grade fluid. Fluid flow is induced by a linear stretching sheet. The sheet is stretched by two equal and opposite forces with the velocity $U_w(x) = ax$. The x- and y-axes are taken along and perpendicular to the sheet respectively and the flow is confined to $a \geq 0$. The sheet is kept at constant temperature T_w whereas T_∞ being the ambient temperature such that $T_w > T_\infty$. Mass transfer analysis is carried out in the presence of chemical reaction. In the absence of thermal radiation and viscous dissipation, the boundary layer equations governing the flow can be expressed as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial \mathrm{v}}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + \mathrm{v}\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1^*}{\rho}\left[u\frac{\partial^3 u}{\partial y^2 \partial x} + \mathrm{v}\frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + 3\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y}\right] + 2\frac{\alpha_2^*}{\rho}\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} + 6\frac{\beta_3^*}{\rho}\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + \mathrm{v}\frac{\partial T}{\partial y}\right) = -\nabla \cdot \mathbf{q}, \tag{3}$$

$$u\frac{\partial C}{\partial x} + \mathrm{v}\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2} - k_m(C - C_\infty). \tag{4}$$

The corresponding boundary conditions are

$$u = U_w(x) = ax, \mathrm{v} = 0, \quad T = T_w, \quad C = C_w \quad \text{at} \quad y = 0,$$

$$u \to 0, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{as} \quad y \to \infty, \tag{5}$$

where u and v represent velocity components along the x- and y-directions respectively, ν the kinematic viscosity, α_1^*, α_2^* and β_3^* the material parameters, ρ the fluid density, c_p the specific heat, $\mathbf{q}$ the heat flux, C the concentration, C_w the fluid wall concentration, C_∞ the ambient fluid concentration, D_m the diffusion coefficient and k_m the first order chemical reaction parameter. Following [3] the heat flux $\mathbf{q}$ is given by

$$\mathbf{q} + \lambda\left(\frac{\partial \mathbf{q}}{\partial t} + \mathbf{V}\cdot\nabla\mathbf{q} - \mathbf{q}\,.\nabla\mathbf{V} + (\nabla.\mathbf{V})\mathbf{q}\right) = -k\nabla T, \tag{6}$$

in which λ is the thermal relaxation time and k the thermal conductivity of fluid. Note that Eq. (6) simplified to Fourier's law for $\lambda = 0$. Since the fluid is incompressible so $\nabla.\mathbf{V} = 0$ and we have

$$\mathbf{q} + \lambda\left(\frac{\partial \mathbf{q}}{\partial t} + \mathbf{V}\cdot\nabla\mathbf{q} - \mathbf{q}\,.\nabla\mathbf{V}\right) = -k\nabla T. \tag{7}$$

Eliminating $\mathbf{q}$ between Eqs. (3) and (7), we obtain following governing equation:

$$u\frac{\partial T}{\partial x} + \mathrm{v}\frac{\partial T}{\partial y} + \lambda\left(u^2\frac{\partial^2 T}{\partial x^2} + \mathrm{v}^2\frac{\partial^2 T}{\partial y^2} + \left(u\frac{\partial u}{\partial x} + \mathrm{v}\frac{\partial u}{\partial y}\right)\frac{\partial T}{\partial x} + 2u\mathrm{v}\frac{\partial^2 T}{\partial x \partial y} + \left(u\frac{\partial v}{\partial x} + \mathrm{v}\frac{\partial v}{\partial y}\right)\frac{\partial T}{\partial y}\right) = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}. \tag{8}$$

We employ the following transformations

$$\eta = \sqrt{\frac{a}{\nu}}y, \quad \Psi = x\sqrt{\nu a}f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \tag{9}$$