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A hybrid approach for the sensitivity analysis of integrated inductors

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ABSTRACT

This paper proposes a hybrid methodology for the evaluation of integrated inductors sensitivity against technological/geometrical parameters variation. The obtained results are used in an optimization-based design environment for integrated inductors, as a way of guaranteeing that obtained solutions are robust against parameter variation. For the inductor characterization, a lumped element model is used, where each element value is evaluated through physics based equations. The sensitivity of the inductor characterization to parameter variations is evaluated at two levels. At the physical level, the sensitivity of the model element values to technological/geometrical parameters variations is computed through an equation-based strategy. Then, the sensitivity of the inductor characterization to the model parameter variations is obtained through a simulation-based approach, where the Richardson extrapolation technique is used for the calculation of the partial derivatives. Several examples considering the evaluation of sensitivity of both inductance and quality factor of two inductors in UMC130 technology are presented. Obtained results are compared against Monte-Carlo simulations.

1. Introduction

Current CMOS technologies have made possible the development of fully integrated wireless communication systems operating in the radio-frequency (RF) range. With the use of CMOS technologies that are approaching the nano-scale, deviations in the circuit behavior due to parameter variations become more significant. Generally, in nano-CMOS technology, imperfections in analog and digital circuits are commonly referred as process parameter variability. Parameter variability is a consequence of several physical processes which occur during fabrication, such as line edge roughness, random dopant fluctuations and oxide thickness variations [1–4]. Since parameter variability has strong influence on circuit reliability as well as in the circuit lifetime, designers are usually tempted to use large design margins, degrading the circuit performance. In order to overcome these issues, reliable assessments must be done at the design stage [1]. The need for obtaining variation tolerant physical designs forces the adoption of new design methodologies, where both parasitics and parameter variability are accounted for. The analysis of the circuit robustness concerning the parameter variability is often called *sensitivity*.

Among the several wireless communication systems blocks, voltage controlled oscillators (VCOs) [5–9], low-noise amplifiers (LNAs) [10–12] and filters make extensive use of integrated inductors. LC-VCOs are fundamental blocks in today's communications systems, and they are responsible for carrier frequency synthesis to up-convert and down-convert signals. The stability of the output frequency is of paramount importance for guarantying the quality and performance of information transfer, and is measured by the VCO phase-noise [9]. For the minimization of the phase-noise, high-quality LC-tanks must be used (where the losses associated to the integrated inductor, for the inductance values needed, dominate those from the capacitor/varactor). Thus, the biggest challenge relies on the design of integrated inductors with maximum quality factor for RF operating frequencies [13].

For the design of RF-integrated circuits, designers usually choose an inductor design from a rather limited set of predefined solutions offered by each specific technology. Optimization of the overall circuit is then performed for an already fixed inductor, hindering the designer the possibility for taking full advantage of alternative inductor geometries. Inductor optimization, on the other hand, may be considered at the first stage of the overall design. This process may be obtained either through simulation-based or model-based methodology. Simulation-based optimization of inductors will compromise the efficiency of the overall design time since electromagnetic simulations are extremely time consuming. To overcome this limitation significant effort has been employed in investigating inductor models. Several lumped

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element models have been proposed as a way of accounting for parasitic effects, such as the metal resistance or the capacitance across the oxide, among others.

This paper addresses the optimization based-design of integrated inductors where the robustness of the obtained designs is assessed through the evaluation of the sensitivity of the inductor quality factor against technological parameter variation. In Section 2 a brief introduction to sensitivity evaluation is given. In Section 3 the two particular cases for the evaluation of integrated inductors quality factor sensitivity against technological parameters variation are considered. For the determination of the inductor quality factor a model-based approach is adopted. Firstly, the simple π-model is considered and then the double-π model will be taken into account. The need for using a hybrid approach for the sensitivity analysis when the double π-model is considered will be highlighted. A set of working examples is presented in Section 4. Finally, conclusions are driven in Section 5.

2. Sensitivity analysis

The sensitivity of a circuit is defined as its capacity to react with changes in certain parameters. It is a measure of how the circuit responds to an undesirable parameters variation. Considering a circuit performance function defined by $F(s)$, then, the sensitivity $Sen_x^{F(s)}$ is defined to be the relative variation of $F(s)$ with respect to the variation of a circuit parameter, x, and is analytically defined by (1).

$$Sen_x^{F(s)} = \frac{x}{F(s)} \cdot \frac{\partial F(s)}{\partial x} \tag{1}$$

For the evaluation of (1) either an analytical approach may be used, or, in such cases where no analytical formulation is possible, simulation-based procedures may be adopted [14].

2.1. Analytical sensitivity analysis

The analytical sensitivity approach relies on the direct application of (1) and can be adopted when circuit performance can be characterized by a differentiable function, $F(s)$. This approach has the advantages of being timely effective and generating accurate results, i.e., without numerical approximation errors.

For more complex functions, where the relation between $F(s)$ and the parameter x_k is not easily differentiable, (2) may be employed [15].

$$Sen_{x_k}^{F(s)} = \sum_{i=1}^{m} Sen_{a_k^i}^{F(s)} \cdot Sen_{x_k}^{a_k^i} \tag{2}$$

For circuits where a large number of variables must be considered, and the explicit dependence of variables is not easily computable, the strategy adopted in recent publications is a symbolic calculation method based on binary decision diagram or graph-pair decision diagram [15–17].

In some circuits, however, it is not possible to derive a single analytical expression for the characterization of a pre-defined performance function. In such cases, conventional analytical sensitivity can no longer be applied; simulation-based techniques should be considered [14].

2.2. Sensitivity analysis by applying Richardson extrapolation

When no analytical expression for the circuit performance function can be easily obtained, then numerical approximation techniques may be applied for the evaluation of the partial derivatives in (1). In this work an approximation technique for computing the partial derivatives called the Richardson extrapolation [18,19] is considered. The Richardson extrapolation method is an evolution acceleration process used to improve the rate of convergence of calculation of the partial derivatives. This method considers (3) for the evaluation of the derivative

$$\frac{\partial F_i}{\partial x_j} \cong \frac{F_i(x_1, ..., x_j+h, ..., x_n) - F_i(x_1, ..., x_j, ..., x_n)}{h} \tag{3}$$

where h is a finite value.

In (3), h is a step parameter that is updated in each iteration. For this case $h_u = 2^{-u}\, h_{u-1}$, where u is the current iteration and h_0 is assigned to an initial value [18,19]. Our proposed sensitivity analysis approach is based on the Richardson extrapolation by applying the simulation-based evaluation outlined in Algorithm 1 [19]

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Algorithm.1. Richardson extrapolation pseudo-code.

```
$h=h_0$
for $v=0;\ v<n;\ v++$ do
  for $u=0;\ u<n;\ u++$ do
    if $u=0$ then
      $f_1$=Simulation-based evaluation with the parameter $x_j+h$
      $f_2$=Simulation-based evaluation with the parameter $x_j$
      Calculate $S[v][u]=(f_1-f_2)/h$
    else
      $S[v][u]=S[v][u-1]+(S[v][u-1]-S[v-1][u-1])/\ldots$
                              ((2\widehat(2 u))−1)
    end if
    $h=h/2.0$
  end for
end for
return $S[v][u]$
```

3. Integrated inductor sensitivity analysis

For the generation of the inductor characteristics, i.e., inductance, L, and quality factor, Q, inductor models are considered. In the first case, the single π-model is addressed. Since for this model, fully analytical expressions may be obtained for both the inductance and the quality factor, a fully analytical approach is adopted for the evaluation of the sensitivities. For the double π-model, no analytical expressions are available for the evaluation of either the inductance or the quality factor. In this case, a mixed simulation based/analytical methodology is considered.

Integrated inductors show three major sources of losses that must be accounted for. Namely, the series resistance of the inductor, which depends on geometric and technological parameters, such as the inductor length, Eddy currents and the skin effect at high frequencies; the capacitive coupling between metal and substrate; and the power losses due to Eddy currents in the substrate.

In the next subsections both models and the corresponding approach for evaluating sensitivities are addressed.

3.1. Sensitivity analysis for the single π-model

Due to its simplicity, as well as the high accuracy of results for frequencies of operation in the order of a few GHz, the single π-model, presented in Fig. 1, is widely used [20]. That model represents not only the spiral inductance and resistance, but also the parasitic elements; L_s and R_s represent the inductance and resistance of the spiral, respectively. The narrow proximity between the inductor tracks and underpass, forms a capacitive