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Polarized scattering by Gaussian random particles under radiative torques

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ABSTRACT

We study the internal alignment of a statistical ensemble of Gaussian random ellipsoids with respect to the radiation direction. We solve the rigid body dynamics due to scattering forces and torques, using a numerically exact and efficient *T*-matrix solver for arbitrary particle shapes and compositions. We then compare the polarization of the aligned ensemble to a randomly oriented ensemble and a perfectly aligned ensemble. We find that the ensemble becomes partially aligned under monochromatic radiation and that the internal alignment has an significant effect on the intensity and polarization of the scattered light.

1. Introduction

Understanding the statistical behavior of dust is crucial in interpretation of observational results and in the tackling of inverse problems, such as deducing magnetic field properties from observed polarization [1] or dust properties in general.

Aligned dust particles were shown to be the cause of interstellar polarization in the near-infrared and the visible light regimes in 1949 independently by Hall and Hiltner [2,3]. In a few years, the discussion about the causes of alignment were started by Davis and Greenstein [4]. In the following several decades, the dominating mechanism of alignment was debated, until radiative torques became the leading explanation to observations, with other effects contributing in different local environments [5,6].

Even though the subtle interactions causing many local abnormalities in the observation data are understood better than ever (see current state in [7,8] and references therein), much groundwork in understanding the observations can still be done. For example, statistical modeling of the effect of dust dynamics on polarization has been an unreachable computational effort until recent years.

In this work, our aim is to illustrate the effect of scattering of light from dust particles to the polarization of dust using state-of-the-art numerical scattering methods. We focus on the bare problem on the effect of scattering only to the dynamics, leading the way to addition of several physical processes found in the interstellar environment, e.g. gas bombardment, Larmor precession and paramagnetic relaxation [7]. We model solid dust particles using Gaussian random shapes [9,10]. The reaction of the particles to the scattering of different wavelengths is studied by numerically integrating the equations of motion. The results are then used to create an ensemble average of the scattering matrix describing angle-dependent intensity and polarization.

To the best knowledge of the authors, such numerically exact methods have never been applied in the same scale for dynamical systems.

2. Theory of scattering dynamics

In this work, the combination of rigid body dynamics, electromagnetic scattering, and radiative forces and torques are shortened as scattering dynamics. In scattering dynamics, we will solve the equations of motion for a dust particle through direct step-by-step calculations. This is possible for an arbitrary particle through a fast and accurate way of solving the *T*-matrix of scattering.

In the following subsections, the relevant physics in scattering dynamics is reviewed.

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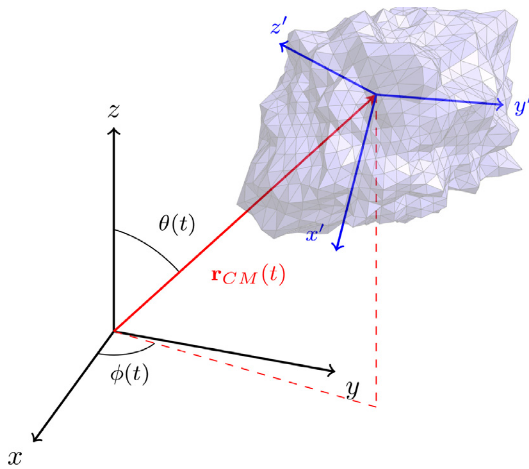

Fig. 1. The position of a tetrahedral model of a solid particle with its principal axes with respect to the laboratory coordinates. Each tetrahedron is handled with respect to the principal axis coordinates. Orientation can be handled in the plane wave case separately with e.g., using rotation matrices, describing the orientation of the principal axes w.r.t. to the laboratory frame axes.

2.1. Dynamics of a rigid body

A rigid body can be used to model a dust particle in almost any conceivable situation. Even in such situations, where a real dust particle would deform or break, the change in the inertia parameters of the particle can be modeled. These situations would, of course, change also the corresponding scattering problem so that methods introduced later would face considerable problems. For this, and the simplicity of notation, we will focus on the theory of a strictly rigid body in our model.

The particle may be a single solid particle, such as illustrated in Fig. 1, or an aggregate. For the purposes of the scattering solver, the solid particles are discretized as tetrahedral meshes, where each tetrahedron is homogeneous. In both cases, the inertia parameters of the particle are solved using the parallel axis theorem. Method of reference tetrahedra [11] simplifies the calculation of the moment of inertia tensor of an arbitrary tetrahedron.

Diagonalization of the moment of inertia tensor gives the so-called principal axes of the particle. Principal coordinates are a type of body coordinates who coincide with the principal axes. Thus, the principal coordinate system is defined with respect to the laboratory coordinates by an orientation matrix

$$\mathbf{P} = \begin{pmatrix} P_{1,x} & P_{2,x} & P_{3,x} \\ P_{1,y} & P_{2,y} & P_{3,y} \\ P_{1,z} & P_{2,z} & P_{3,z} \end{pmatrix} = (\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3), \tag{1}$$

where column vectors $\mathbf{a}_i$ are the principal axes of the particle from smallest moment of inertia to largest. The equations of rotational motion are simplified into Euler's equations in the principal axes, and they are the usual choice for solving rotational dynamics.

2.2. Electromagnetic forces and torques

For a general description of scattering, the most important quantities are the size parameter of the particle, $x = ka = 2\pi a/\lambda$, the shape and the complex index of refraction $n = n_{Re} + in_{Im}$ of the particle. Above, a is the equivalent radius of the particle to a sphere of the same volume, k is the wavenumber, and λ the wavelength of the incident radiation. Usually, the dimensionless size parameter is enough to describe many interesting quantities. However, in order to make sense of the dynamical time scales, it is temptating to choose a concrete size and density for the particle and fix the wavelengths to correspond to certain size parameters.

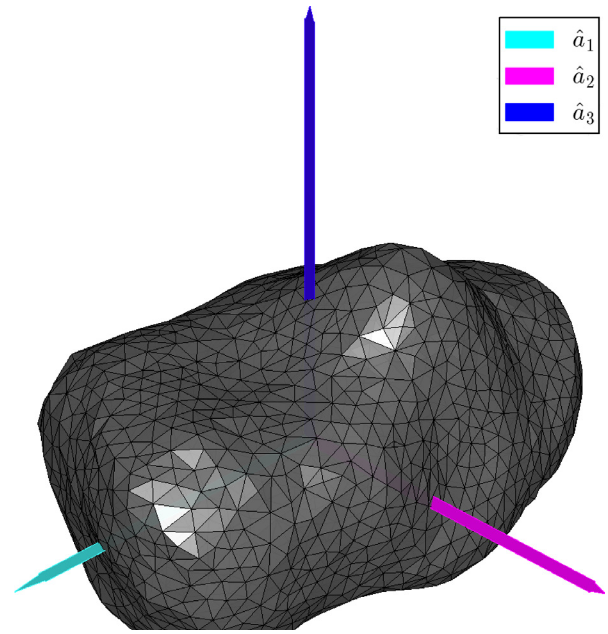

Fig. 2. An example of a Gaussian random ellipsoid, represented as a tetrahedral mesh, with $\sigma = 0.125$, $l = 0.35$ and $a : b : c = 1 : 0.8 : 0.6$.

In space environments, the incident radiation from starlight is mostly visible and infrared light, and can be modeled as plane waves. Regarding dust in space, a wavelength range of 200–2000 nm corresponds to size parameter range 0.03–30 for particles ranging from 0.01 μm to 1 μm in equivalent radius.

The mechanical effects of radiation are described by the Maxwell stress tensor, T, which should not be confused with the *T*-matrix. The Maxwell stress tensor has components

$$T_{ij} = \varepsilon_0\left(E_iE_j - \frac{1}{2}\delta_{ij}E^2\right) + \frac{1}{\mu_0}\left(B_iB_j - \frac{1}{2}\delta_{ij}B^2\right). \tag{2}$$

For almost all intents and purposes the term with the Maxwell stress tensor dominates the total force in a volume V,

$$\mathbf{F} = \oint_S \mathsf{T}\cdot\hat{\mathbf{n}}\mathrm{d}S - \varepsilon_0\mu_0\int_V \frac{\partial}{\partial t}\mathbf{S}\mathrm{d}V, \tag{3}$$

where S is the surface of V, where momentum transfer is occuring. This is due to the fact, that the latter term describes the momentum contained within the volume instead of being transferred into it. The latter term, containing the Poynting vector $\mathbf{S}$, varies with the frequency of the radiation, and thus for most applications, will be averaged out of consideration [12].

After averaging, total force and torque will be represented by simple surface integrals containing the Maxwell stress tensor,

$$\begin{aligned} \mathbf{F} &= \oint_S \mathsf{T}\cdot\hat{\mathbf{n}}\,\mathrm{d}S, \\ \mathbf{N} &= \oint_S \mathbf{r}\times(\mathsf{T}\cdot\hat{\mathbf{n}})\,\mathrm{d}S. \end{aligned} \tag{4}$$

The torque obtained by solving the scattering problem can be written in terms of normalized quantities [13] as

$$\mathbf{N} = \frac{\lambda a^2}{2c}\langle S\rangle_{\mathrm{inc}}\mathbf{Q_N}, \tag{5}$$

where $\langle S\rangle_{\mathrm{inc}}$ is the incident Poynting vector, and $\mathbf{Q_N}$ is the normalized quantity, the torque efficiency. In particular the torque efficiency can be used to compare results between different geometries with the same consistency.