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Computer simulation of position and maximum of linear polarization of asteroids

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ABSTRACT

The ground-based observations of near-Earth asteroids at large phase angles have shown **some** feature: the linear polarization maximum position of the high-albedo E-type asteroids shifted markedly towards smaller phase angles ($\alpha_{\max} \approx 70°$) with respect to that for the moderate-albedo S-type asteroids ($\alpha_{\max} \approx 110°$), weakly depending on the wavelength. **To study this phenomenon, the theoretical approach and the modified T-matrix method (the so-called Sh-matrices method) were used.** Theoretical approach was devoted to finding the values of $\alpha_{\max}$, corresponding to maximal values of positive polarization $P_{\max}$. Computer simulations were performed for an ensemble of random Gaussian particles, whose scattering properties were averaged over with different particle orientations and **size** parameters in the range X = 2.0 ... 21.0, with the power law distribution X^{-k}, where $k = 3.6$. The real parts of the refractive index m_r were 1.5, 1.6 and 1.7. Imaginary part of refractive index varied from $m_i = 0.0$ to $m_i = 0.5$. Both theoretical approach and computer simulation showed that the value of $\alpha_{\max}$ strongly depends on the refractive index. The increase of m_i leads to increased $\alpha_{\max}$ and $P_{\max}$. In addition, computer simulation shows that the increase of the real part of the refractive index reduces $P_{\max}$. **Whereas E-type high-albedo asteroids have smaller values of m_i, than S -type asteroids, we can conclude, that value of $\alpha_{\max}$ of E-type asteroids should be smaller than for S –type ones. This is in qualitative agreement with the observed effect in asteroids.**

1. Introduction

Near-Earth asteroids (NEA) provide good opportunity to get the maximal complete (for ground-based observations) phase dependence of the polarization including the value and position of the maximum degree of polarization ($P_{\max}$,$\alpha_{\max}$). It is of great interest to determine the optical and physical characteristics of particles in the regolith layer of asteroids. First of all, using **empirical** relation $\log p_v = -0.71 \log P_{\max} - 1.63$, which reflects the relationship between the geometric albedo p_v of the surface and the maximum polarization, one can obtain albedo of the surface (see, e.g. [1]). However, polarimetric observations of NEA at large phase angles are still very rare and do not cover the entire main types of asteroids. At the present time the polarization curves for wide phase angle range were obtained only for S- and E- type asteroids. They are presented in Fig. 1.

Observations confirm the phenomenological Umov effect [4]. The maximum degree of polarization E-type asteroids was much lower ($P_{\max} = 1.7 \pm 0.2\%$) then ($P_{\max} = 8.1 \pm 0.2\%$) for S-type asteroids in the V-band [5,6]. But the **unexpected** feature was that the polarization maximum position for E-type asteroids was significantly shifted towards small phase angles ($\alpha_{\max} = 71 \pm 10°$) in comparison with ($\alpha_{\max} = 110 \pm 10°$) for S-type asteroids. The effect of particle properties on the polarization characteristics of scattered light was studied in a number of papers (see, e.g. [7–11]). However, in these works, the difference in the position of the maximum polarization of different asteroids was not explicitly studied. This paper is devoted to the study of this observed effect. **We focused on the influence of real and imaginary parts of the refractive index on the linear polarization maximum. Certainly, the polarization maximum and its position also depend on the sizes and structure of regolith particles, this is a topic for further research.**

In this paper we found an analytical expressions (using Hapke's approach) and carried out a computer simulation. Theoretical approach is described in Section 2. Section 3 **is devoted to computer simulation and description of its results.** Section 4 **summarizes the main results of paper.**

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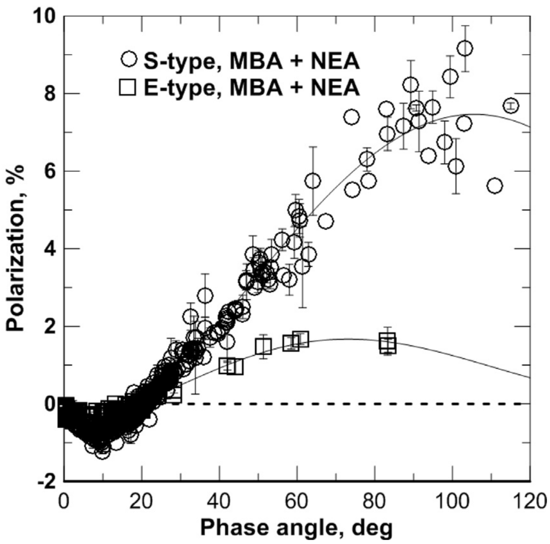

Fig. 1. Composite phase dependencies of the degree of polarization of moderate-albedo S-type asteroids (circles) and high-albedo E-type asteroids (squares) in the V band constructed from Asteroid Polarimetric Database V8.0. EAR-A-3-RDR-APD-POLARIMETRY-V8.0. NASA Planetary Data System [2]. Curves represent the approximation of data by a trigonometric expression [3].

2. Theoretical approach

As is known, positive branch of the linear polarization is controlled primarily by the properties of the individual particles of the medium [12]. **So, it is possible to calculate the effects of macroscopic roughness on light scattered by a surface having an arbitrary diffuse reflectance function [**12**]. A theoretical expression for *P* may be derived as follows.** The positive branch of linear polarization degree of light, scattered by particulate media with albedo ω, refractive index of particles $m_0 = m_r + i \bullet m_i$ in the case of the geometry of illumination/observation, where $i = \varepsilon = \frac{\alpha}{2}$ **(note, that the geometry of illumination/observation can be any, but we choose the above for simplicity**) and macroscopic roughness, characterized by a mean roughness slope angle $\bar{\theta}$, can be described by following equation:

$$P(\alpha) = \frac{1}{2} \cdot \frac{R_\perp\left(\frac{\alpha}{2}\right) - R_{||}\left(\frac{\alpha}{2}\right)}{R_\perp\left(\frac{\alpha}{2}\right) + R_{||}\left(\frac{\alpha}{2}\right) + \omega - S_e + \omega\left[H^2\left(\frac{\mu(\alpha)}{K}\right) - 1\right]}, \tag{1}$$

where α is the phase angle, S_e is the total fraction of the light externally incident on the surface of the particle that is specularly reflected, $R_\perp$ and $R_{||}$ are Fresnel reflection coefficients, **which describe two polarizations of light** [13], *H(x)* is the Ambartsumian–Chandrasekhar H function, ω is albedo, K is the porosity coefficient, $\mu = \mu_e = \mu_{0e}$is the cosine of effective angle of incidence/emergence (**angle of incidence and angle of emergence on tilted area with the same phase angle [**12**]**). All functions from Eq. (1) are described in book [12] in details.

In what follows, we will need the derivatives of the Fresnel coefficients, which can be represented in the form:

$$R'_\perp(\alpha) = \frac{\partial R_\perp(\alpha)}{\partial\alpha} = 2\frac{(m_r - \cos(\alpha))\sin(\alpha)}{(m_r + \cos(\alpha))^2 + m_i^2} + 2\frac{(m_r + \cos(\alpha))\sin(\alpha)\left[(m_r - \cos(\alpha))^2 + m_i^2\right]}{\left[(m_r + \cos(\alpha))^2 + m_i^2\right]^2}, \tag{2}$$

$$R'_{||}(\alpha) = \frac{\partial R_{||}(\alpha)}{\partial\alpha} = -2\frac{\left(m_r - \frac{1}{\cos(\alpha)}\right)\sin(\alpha)}{\cos^2(\alpha)\left[\left(m_r + \frac{1}{\cos(\alpha)}\right)^2 + m_i^2\right]} - 2\frac{\left(m_r + \frac{1}{\cos(\alpha)}\right)\sin(\alpha)\left[\left(m_r - \frac{1}{\cos(\alpha)}\right)^2 + m_i^2\right]}{\cos^2(\alpha)\left[\left(m_r + \frac{1}{\cos(\alpha)}\right)^2 + m_i^2\right]^2}. \tag{3}$$

Moreover, Ambartsumian–Chandrasekhar H function is the solution of the integral equation [14]:

$$H(x) = 1 + \frac{\omega}{2}xH(x) \cdot \int_0^1 \frac{H(x')}{x + x'}dx'. \tag{4}$$

Using an excellent approximation of *H*(*x*), **taken from** [12]:

$$H(x) \approx \left\{1 - \frac{\omega}{2}x\int_0^1 \frac{(1+2r_0x')}{x+x'}dx'\right\}^{-1} = = \left\{1 - \omega x\left[r_0 + \frac{1-2r_0x}{2}\ln\left(\frac{1+x}{x}\right)\right]\right\}^{-1} \tag{5}$$

where $r_0 = (1-\gamma)/(1+\gamma)$ and $\gamma = \sqrt{1-\omega}$**.** Then we obtain the derivation of Ambartsumian–Chandrasekhar function, which is very convenient for practical calculations:

$$H'(x) = \frac{\partial H(x)}{\partial x} = \omega\frac{r_0 + \frac{1-2r_0x}{2}\ln\left(\frac{1+x}{x}\right) + x\left[-r_0\ln\left(\frac{1+x}{x}\right) + \left(\frac{1}{2} - r_0x\right)\left(\frac{1}{1+x} - \frac{1}{x^2}\right)\right]}{\left\{1 - \omega x\left[r_0 + \frac{1-2r_0x}{2}\ln\left(\frac{1+x}{x}\right)\right]\right\}^2}, \tag{6}$$

Values of the parameters used for calculation, was $S_e = 0.0587 + 0.8543R(0) + 0.087R(0)^2$**, where** $R(0) = R_\perp(0) = R_{||}(0)$ **- Fresnel reflection coefficients at zero phase angle, $K = 1$ and** $\omega = 1/(1 + Xm_i)$ [12]**. X is the size parameter, which was chosen to be $X = 60$.**

Fig. 2 shows the phase dependences of linear polarization, calculated according to the equations from [12], for different sets of parameters: real part of refractive index is equal $m_r = 1.5$ (left panel), $m_r = 1.6$ (middle panel) and $m_r = 1.7$ (right panel), correspondingly.

One can see that the positive polarization maximum shifts when imaginary part of refractive index m_i is changed. Can this shift be described analytically?

To derive where the maximum of linear polarization α_{max} is, we should find the roots of following equation:

$$\left.\frac{\partial P}{\partial\alpha}\right|_{\alpha=\alpha_{max}} = 0. \tag{7}$$

If we substitute Eq. (1) into (7), finally we obtain the following equation:

$$\frac{R'_\perp\left(\frac{\alpha}{2}\right) - R'_{||}\left(\frac{\alpha}{2}\right)}{R_\perp\left(\frac{\alpha}{2}\right) + R_{||}\left(\frac{\alpha}{2}\right)} = \frac{R'_\perp\left(\frac{\alpha}{2}\right) - R'_{||}\left(\frac{\alpha}{2}\right) + 4\omega H'(\mu(\alpha))H(\mu(\alpha))\mu'(\alpha)}{R_\perp\left(\frac{\alpha}{2}\right) + R_{||}\left(\frac{\alpha}{2}\right) + \omega - S_e + \omega[H^2(\mu(\alpha)) - 1]}, \tag{8}$$

where

$$\mu'(\alpha) = \frac{\partial\mu(\alpha)}{\partial\alpha} = = \frac{\chi(\bar{\theta})}{2} \cdot \left[-\sin\left(\frac{\alpha}{2}\right) + \cos\left(\frac{\alpha}{2}\right)\tan\left(\bar{\theta}\right)\frac{\cos(\alpha)E_2\left(\frac{\alpha}{2}\right) + \sin^2\left(\frac{\alpha}{2}\right)E_2\left(\frac{\alpha}{2}\right)}{2 - E_1\left(\frac{\alpha}{2}\right) - \frac{\alpha}{\pi}E_1\left(\frac{\alpha}{2}\right)} + \right. + \sin\left(\frac{\alpha}{2}\right)\tan\left(\bar{\theta}\right)\frac{\left[-2\sin(\alpha)E_2\left(\frac{\alpha}{2}\right) + \cos(\alpha)E'_2\left(\frac{\alpha}{2}\right) + \sin(\alpha)E_2\left(\frac{\alpha}{2}\right)\right]}{2 - E_1\left(\frac{\alpha}{2}\right) - \frac{\alpha}{\pi}E_1\left(\frac{\alpha}{2}\right)} - \left. - \sin\left(\frac{\alpha}{2}\right)\tan\left(\bar{\theta}\right)\frac{\left[\cos(\alpha)E_2\left(\frac{\alpha}{2}\right) + \sin^2\left(\frac{\alpha}{2}\right)E_2\left(\frac{\alpha}{2}\right)\right]\cdot\left(E'_1\left(\frac{\alpha}{2}\right) - \frac{2}{\pi}E_1\left(\frac{\alpha}{2}\right) - \frac{\alpha}{\pi}E'_1\left(\frac{\alpha}{2}\right)\right)}{\left[2 - E_1\left(\frac{\alpha}{2}\right) - \frac{\alpha}{\pi}E_1\left(\frac{\alpha}{2}\right)\right]^2}\right], \tag{9}$$

where

$$\chi\left(\bar{\theta}\right) = \left\langle\cos\left(\bar{\theta}\right)\right\rangle = \frac{1}{\sqrt{1 + \pi\tan\left(\bar{\theta}\right)^2}} \tag{10}$$

and