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Variable discrete ordinates method for radiation transfer in plane-parallel semi-transparent media with variable refractive index

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ABSTRACT

The traditional form of discrete ordinates method is applied to solve the radiative transfer equation in plane-parallel semi-transparent media with variable refractive index through using the variable discrete ordinate directions and the concept of refracted radiative intensity. The refractive index are taken as constant in each control volume, such that the direction cosines of radiative rays remain non-variant through each control volume, and then, the directions of discrete ordinates are changed locally by passing each control volume, according to the Snell's law of refraction. The results are compared by the previous studies in this field. Despite simplicity, the results show that the variable discrete ordinate method has a good accuracy in solving the radiative transfer equation in the semi-transparent media with arbitrary distribution of refractive index.

1. Introduction

In recent years, much attention has been paid to solve the radiative transfer equation (RTE) in semi-transparent media (STM), because of widespread applications in industries, such as protecting coating, waveguide materials, optical measurement of flames, glass production and so on. As the radiative rays travel in curve paths, the analysis of radiative heat transfer in the STM with variable refractive index is more complicated than that in those participating media with uniform refractive index where the radiative rays travel on straight paths.

Many investigations have been made to present an efficient approach to solve the radiation transfer in the STM. Some of the earliest works on this field were presented by Siegel and Spuckler [1,2], who proposed a model to solve the RTE in one-dimensional plane-parallel STM by dividing the composite medium into several sublayers each at uniform refractive index bounded by diffuse surfaces. The radiative transfer problem in semi-transparent graded index media were solved through curved ray-tracing techniques by Ben Abdallah et al. [3–6], Huang et al. [7,8], and Liu and coworkers [9–11]. Since the curved ray tracing is complicated, some approaches were developed to trace the curve paths by dividing the medium into incremental slices where the refractive index is essentially assumed constant, and hence, the radiative rays travel straight lines. For example, Krishna and Mishra [12] used the discrete transfer method (DTM) to solve the RTE in plane-parallel STM with linear variation of refractive index. Application of the DTM was improved to solve the RTE in absorbing-emitting STM with arbitrary refractive index distribution by Sarvari [13] who introduced a new quantity, namely the *refracted intensity*, by which the RTE in non-unit or variable refractive index media may be reduced into the regular form of RTE with unit and invariant refractive index.

The discrete ordinate method (DOM), developed by Chandrasekhar [14] and improved during the years by other researchers [15–23], has been widely used because of its simplicity and straightforward essence in solving the RTE. In addition, the DOM is known as the most compatible approach to combine with other control-volume-based methods to solve the combined modes of heat transfer. In the DOM, the medium is divided into infinitesimal control volumes. The center point of each control volume is the source of radiative rays, which propagate into all directions that cover a sphere of solid angles. The directional variation of the radiative intensity is represented by a set of discrete invariant directional ordinates spanning the total solid angle range of 4π. Since the radiative rays propagate along curve paths in the STM, the ordinate directions may change according to the Snell's law of refraction [24]. Lemonnier and Le Dez [25] extended the DOM to solve the RTE in the plane-parallel STM by splitting the streaming operator into two parts to account the spatial and directional variations of the intensity, and then they considered an invariant set of ordinate directions and their associated weights. Despite very good accuracy, this method is restricted to a monotonic variation of the refractive index. This approach were used by Namjoo et al. [26] for solving an inverse problem in the STM.

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Nomenclature

a	anisotropy factor
G	incident radiation, W/m^2
I	radiative intensity, W/m^2sr
J	number of volume elements
M	number of rays propagated from wall surface
n	refractive index
Q	radiative heat flux, W/m^2
R	wall refractive ratio
s	geometric path length, m
T	temperature, K
u,v	weights

Greek symbols

β	extinction coefficient, m^{-1}
χ	direction cosine of edge direction
ε	emissivity
Φ	non-dimensional emissive power
Γ	refracted incident radiation, W/m^2
κ	absorption coefficient, m^{-1}
μ	direction cosine of ordinate direction
θ	polar angle, *rad*
Θ	refracted heat flux, W/m^2
σ	Stefan-Boltzmann constant, W/m^2K^4
σ_s	scattering coefficient, m^{-1}
τ	optical depth
ω	single scattering albedo
Ψ	non-dimensional heat flux
$\Im$	refracted intensity, W/m^2sr

Subscripts

b	blackbody
w	wall

Superscripts

$+$, $-$	into positive and negative direction

In this paper a variable discrete ordinates method (VDOM) is represented to solve the RTE in absorbing-emitting-scattering STM with variable refractive index. In this approach, the regular discrete ordinate form of the RTE remains invariant through using the concept of the refracted intensity [13], but instead, the set of ordinate directions is updated locally to consider the curvature of ray paths, so that the number and direction of ordinates may vary locally. The accuracy of the present method is examined by comparing its results with those obtained by previous studies. Despite simplicity, the VDOM has a good accuracy in solving the RTE in the STM with arbitrary refractive index profile.

2. Governing equations

The RTE and its boundary condition along a pencil of ray in an absorbing-emitting-scattering STM are as follows:

$$\frac{d\Im^{\pm}}{d\tau_s} + \Im^{\pm} = S^{\pm}, \quad 0 \le \pm\mu^{\pm} \le 1 \tag{1a}$$

$$\Im^{\pm}_{w^{\mp}} = \varepsilon_{w^{\mp}} I_{bw^{\mp}} + 2(1-\varepsilon_{w^{\mp}}) \int_0^{\mp 1} \Im^{\mp}_{w^{\mp}}\, \mu^{\mp}_{w^{\mp}}\, d\mu^{\mp}_{w^{\mp}} \tag{1b}$$

where $\Im = I/n^2$ is known as the *refracted intensity*, $\mu = \cos\theta$ is the direction cosine of polar angle, $\tau_s = \beta\, s$ is the optical distance along the direction of ray propagation. Here, superscripts – and + denote the rays emanating in the negative and positive directions, respectively, and $w^{\pm}$ denotes the value on upper/lower wall. The source term, S, in Eq. (1a) is given by

$$S^{\pm} = (1-\omega)I_b + \frac{\omega}{2}\int_{-1}^{+1} \Im^{\pm}\left(\mu_i^{\pm}\right) \varphi\left(\mu_i^{\pm}, \mu^{\pm}\right) d\mu_i^{\pm} \tag{2}$$

where $\omega = \sigma_s/\beta$ and φ are the single scattering albedo and the scattering phase function, respectively.

The *refracted heat flux*, $\Theta = Q/n^2$, and the *refracted incident intensity*, $\Gamma = G/n^2$, are given by

$$\Theta = 2\pi \int_{-1}^{1} \Im^{\pm}\mu^{\pm}\, d\mu^{\pm} \tag{3a}$$

$$\Gamma = 2\pi \int_{-1}^{1} \Im^{\pm} d\mu^{\pm} \tag{3b}$$

and the *divergence of refracted heat flux*, $\nabla \cdot Q/n^2$ is obtained by

$$\nabla \cdot \Theta = \kappa\,(4\pi I_b - \Gamma) \tag{4}$$

3. Numerical modeling of the VDOM

In the VDOM, the physical medium between parallel plates is divided into control volumes. The angular domain on each wall surface is divided into equal polar angles. Directions of ordinates at wall surfaces are taken along the central directions of polar angles (see Fig. 1). The directions of ordinates change for the next control volumes according to the Snell's law. The blackbody intensity and the radiative properties of the medium are taken as constant in each control volume. In addition, the medium refractive index is assumed constant in each control volume, such that the radiative rays travel straight lines in control volumes, and in consequence, the direction cosines of radiative rays remain non-variant through each control volume. Hence, the RTE along the ordinate m in control volume j may be written as:

$$\mu^{\pm}_{m,j}\frac{d\Im^{\pm}_{m,j}}{d\tau_j} + \Im^{\pm}_{m,j} = S^{\pm}_{m,j} \tag{5}$$

where $\tau_j = \beta\, z_j$ is the optical thickness along the z-direction, and $\mu^{\pm}_{m,j} = \cos\theta^{\pm}_{m,j}$ is the direction cosine of radiative ray into positive/negative direction.

Integrating Eq. (5) over the incremental optical thickness, $\Delta\tau_j$, leads to the following relation

$$\mu^{\pm}_{m,j}\Im^{\pm}_{m,j^{\pm}} - \mu^{\pm}_{m,j}\Im^{\pm}_{m,j^{\mp}} + \Im^{\pm}_{m,j}\Delta\tau_j = S^{\pm}_{m,j}\Delta\tau_j \tag{6}$$

where the subscript $j^{\pm}$ denotes the upper/lower interface of control volume (see Fig. 1a). The refracted intensity at each interface may be related to the refracted intensity at center of control volume by a linear relationship as

$$\Im^{\pm}_{m,j} = \alpha\, \Im^{\pm}_{m,j^{\pm}} + (1-\alpha)\, \Im^{\pm}_{m,j^{\mp}} \tag{7}$$

where $1/2 \le \alpha \le 1$ is the weighting factor. Using Eq. (7), the unknown refracted intensities, $\Im^{\pm}_{m,j^{\pm}}$, can be eliminated from Eq. (6), and the refracted intensities at the center of control volumes are evaluated by

$$\Im^{\pm}_{m,j} = \frac{S^{\pm}_{m,j}\Delta\tau_j \pm \Im^{\pm}_{m,j^{\mp}}\mu^{\pm}_{m,j}/\alpha}{\Delta\tau_j \pm \mu^{\pm}_{m,j}/\alpha}, \quad 0 \le \pm\mu^{\pm} \le 1 \tag{8}$$

A radiation ray redirects as passing the control volume interface according to the Snell's law of refraction

$$n_j\left[1 - (\mu^{\pm}_{m,j})^2\right]^{1/2} = n_{j\mp 1}\left[1 - (\mu^{\pm}_{m,j\mp 1})^2\right]^{1/2} \tag{9}$$

and therefore, the direction of associated discrete ordinate may changes accordingly. The radiative ray specularly reflects when the *reflection criterion* is satisfied. This criterion is given by

$$\left[1 - (\mu^{\pm}_{m,j})^2\right] > 1 \tag{10}$$