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Scattering of the evanescent components in the angular spectrum of a tightly focused electromagnetic beam by a spherical particle

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A B S T R A C T

In many previous studies where a tightly focused beam is scattered by a spherical particle, either the experimental conditions were such that evanescent components were absent from the angular spectrum of the incident beam, or if they were present, it was assumed that their contribution to scattering was small with respect to that of the oscillatory components, and could safely be ignored. In this paper the contribution of the evanescent components is explicitly calculated and the validity of their neglect in various situations is assessed. It is demonstrated that when a particle whose size is comparable to the wavelength is located near the plane in which the strength of the evanescent components is maximized, the angular spectrum components just inside the evanescent regime, when added to the contribution of the oscillatory components, can possibly make a significant contribution to the shape coefficients of the incident beam.

1. Background, notation, and motivation

The angular spectrum of a tightly focused beam in general contains both oscillatory and evanescent components. This paper studies the contribution of the evanescent components to scattering by a spherical particle placed in the beam. But before the specific goals of this paper can be clearly stated, a certain amount of context must be given. When a monochromatic electromagnetic beam propagating in the $+z$ direction and having wavelength λ and wave number $k=2\pi/\lambda$ is incident on a spherical particle of radius a and refractive index N, the interaction of the beam with the particle produces outgoing scattered waves. The fields of the incident beam can be written [1] as a sum of transverse electric (TE) and transverse magnetic (TM) spherical multipole waves of the first type, $\mathbf{M}^{(1)}_{n,m}(k\mathbf{r})$ and $\mathbf{N}^{(1)}_{n,m}(k\mathbf{r})$,

$$\mathbf{E}(\mathbf{r}) = \sum_{n=1}^{\infty}\sum_{m=-n}^{n} i^n(-1)^m C_{n,m}\left[-ig^{\mathrm{TE}}{}_{n,m}\mathbf{M}^{(1)}{}_{n,m}(k\mathbf{r}) - g^{\mathrm{TM}}{}_{n,m}\mathbf{N}^{(1)}{}_{n,m}(k\mathbf{r})\right] \tag{1a}$$

$$c\mathbf{B}(\mathbf{r}) = \sum_{n=1}^{\infty}\sum_{m=-n}^{n} i^n(-1)^m C_{n,m}\left[ig^{\mathrm{TM}}_{n,m}\mathbf{M}^{(1)}_{n,m}(k\mathbf{r}) - g^{\mathrm{TE}}_{n,m}\mathbf{N}^{(1)}_{n,m}(k\mathbf{r})\right], \tag{1b}$$

where the time dependence $\exp(-i\omega t)$ is left implicit, and

$$C_{n,m} = \left\{\pi(2n+1)(n+m)!/[n(n+1)(n-m)!]\right\}^{1/2}. \tag{2}$$

The shape coefficients of the incident beam are $g^{\mathrm{TE}}_{n,m}$ and $g^{\mathrm{TM}}_{n,m}$, n is the integer partial wave number with $1 \le n < \infty$, and m is the azimuthal mode number with $-n \le m \le n$. The factor $C_{n,m}$ in Eq.(2) is chosen so that the $m=1$ beam shape coefficients for a plane wave traveling in the $+z$ direction and linearly polarized in the x direction are $g^{\mathrm{TE}}_{n,1} = -i$ and

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$g^{TM}_{n,1}=1$. The spherical mutipole wave $\mathbf{M}^{(1)}_{n,m}(k\mathbf{r})$ is related to the transverse vector spherical harmonic $\mathbf{X}_{n,m}(\theta,\varphi)$ in the notation of [1] via

$$\mathbf{M}^{(1)}_{n,m}(k\mathbf{r})=j_n(kr)\mathbf{X}_{n,m}(\theta,\varphi), \tag{3}$$

where $j_n(kr)$ is a spherical Bessel function. Similarly, $\mathbf{N}^{(1)}_{n,m}(k\mathbf{r})$ is a combination of the radial and transverse vector spherical harmonics $\mathbf{Y}_{n,m}(\theta,\varphi)$ and $\mathbf{Z}_{n,m}(\theta,\varphi)$, respectively,

$$\mathbf{N}^{(1)}_{n,m}(k\mathbf{r})=\mathrm{i}[n(n+1)]^{1/2}[j_n(kr)/(kr)]\mathbf{Y}_{n,m}(\theta,\varphi)+(1/kr)[krj_n(kr)]'\mathbf{Z}_{n,m}(\theta,\varphi), \tag{4}$$

where the prime symbol indicates the derivative of a function with respect to its argument. The three orthogonal vector spherical harmonics are

$$\mathbf{X}_{n,m}(\theta,\varphi)=[n(n+1)]^{-1/2}\mathbf{L}Y^m_n(\theta,\varphi) \tag{5a}$$

$$\mathbf{Y}_{n,m}(\theta,\varphi)=Y^m_n(\theta,\varphi)\mathbf{u_r} \tag{5b}$$

$$\mathbf{Z}_{n,m}(\theta,\varphi)=\mathbf{u_r}\times\mathbf{X}_{n,m}(\theta,\varphi) \tag{5c}$$

where $\mathbf{L}$ is the angular momentum operator [2] and $Y^m_n(\theta,\varphi)$ are scalar spherical harmonics in the notation of [3]. These in turn are related to associated Legendre functions $P^m_n[\cos(\theta)]$ in the notation of [3] and the azimuthal function $\exp(im\varphi)$. There are many different conventions for associated Legendre functions, all differing from each other by either a constant of proportionality or an occasional minus sign. In this paper the above convention is followed since it straightforwardly generalizes to associated Legendre functions of a complex argument [4].

In generalized Lorenz–Mie theory (GLMT) for scattering of the beam by a spherical particle, the outgoing scattered waves may also be decomposed into a sum of TE and TM spherical multipole waves of the third type, for which the radial function in Eqs. (3) and (4) is the first outgoing spherical Hankel function $h^{(1)}_n(kr)$. The scattering amplitudes $S_1(\theta,\varphi)$ and $S_2(\theta,\varphi)$ are

$$S_1(\theta,\varphi)=\sum_{n=1}^{\infty}\sum_{m=-n}^{n}\{(2n+1)/[2n(n+1)]\}\{-img^{TM}{}_{n,m}a_n\pi^{|m|}_n[\cos(\theta)]+g^{TE}_{n,m}b_n\tau^{|m|}_n[\cos(\theta)]\}\exp(im\varphi) \tag{6a}$$

$$S_2(\theta,\varphi)=\sum_{n=1}^{\infty}\sum_{m=-n}^{n}\{(2n+1)/[2n(n+1)]\}\{img^{TE}_{n,m}b_n\pi^{|m|}_n[\cos(\theta)+g^{TM}_{n,m}a_n\tau^{|m|}_n[\cos(\theta)]\}\exp(im\varphi) \tag{6b}$$

where a_n and b_n are the partial wave scattering amplitudes of Lorenz–Mie theory for scattering by a plane wave propagating in the z direction [5], and $\pi^{|m|}_n[\cos(\theta)]$ and $\tau^{|m|}_n[\cos(\theta)]$ are the angular functions of GLMT [6]. Other scattering quantities of interest, such as the transverse and longitudinal components of the trapping force for laser tweezers applications, contain more complicated combinations of $g^{TM}_{n,m}$, $g^{TE}_{n,m}$, a_n, and b_n [6]. Given the functional form of the fields of the incident beam, the shape coefficients may be calculated most simply by inverting Eqs.(1a) and (1b),

$$g^{TE}_{n,m}=-(-i)^{n+1}(-1)^m(1/C_{n,m})[1/j_n(kr)]\int_0^{\pi}\sin(\theta)\mathrm{d}\theta\int_0^{2\pi}\mathrm{d}\varphi\mathbf{X}^*{}_{n,m}(\theta,\varphi)\cdot\mathbf{E}(r,\theta,\varphi) \tag{7a}$$

$$g^{TM}_{n,m}=(-i)^{n+1}(-1)^m(1/C_{n,m})[1/j_n(kr)]\int_0^{\pi}\sin(\theta)\mathrm{d}\theta\int_0^{2\pi}\mathrm{d}\varphi\mathbf{X}^*{}_{n,m}(\theta,\varphi)\cdot c\mathbf{B}(r,\theta,\varphi), \tag{7b}$$

for which only the transverse components of the beam field contribute to the integrand, and where the asterisk symbol denotes complex conjugation. The kr-dependent prefactor in Eqs. (7a) and (7b) will be canceled by a compensating term resulting from the evaluation of the double integral if $\mathbf{E}(r,\theta,\varphi)$ and $\mathbf{B}(r,\theta,\varphi)$ are an exact solution of Maxwell's equations and the vector wave equation. This insures that the beam shape coefficients are constants. Unfortunately, the functional form of the fields is exactly known for only a few different types of beams, such as a plane wave, a Bessel beam, and a few models of a nominally Gaussian beam. For beams where only approximate functional forms of the fields are known, a residual amount of kr-dependence remains in the beam shape coefficients after the integrations have been performed. One approach to this problem is to evaluate the residual kr-dependence at a convenient location [7,8]. Another approach is to remodel the approximate beam into a very similar beam that is a solution of Maxwell's equations and the vector wave equation [9]. Yet a third approach is to decompose the beam into an angular spectrum of plane waves [10], which also carries out a type of remodeling if the beam model is approximate. The advantage of this third approach is that the beam shape coefficients of each plane wave component in the angular spectrum are analytically known (see Refs. [31–36] of [11] for the history of the evaluation of these coefficients). The total shape coefficients of the incident beam are then the shape coefficients of each plane wave component weighted by their respective amplitudes and phases, and then summed over all the components [12]. This is the approach taken in this paper.

As a historical aside, it should be mentioned in passing that the beam shape coefficients can also be obtained by integrating the scalar product of the radial or transverse components of $\mathbf{E}(r,\theta,\varphi)$ and $\mathbf{B}(r,\theta,\varphi)$ with either [13] $\mathbf{Y}^*_{n,m}(\theta,\varphi)$ or $\mathbf{Z}^*_{n,m}(\theta,\varphi)$, rather than with $\mathbf{X}^*_{n,m}(\theta,\varphi)$ as was done in Eqs. (7a) and (7b). However, the details of the cancellation of the respective kr-dependent prefactor is more complicated for these two alternatives than it is for Eqs.(7a) and (7b). In particular, the details of the cancelation are most elaborate when $\mathbf{Y}^*_{n,m}(\theta,\varphi)$ along with the radial components of the beam fields are used. The key integral required in this approach was evaluated in [14], which corrected an earlier sign error in [15], (see also [16] in this regard). The use of $\mathbf{Y}^*_{n,m}(\theta,\varphi)$ along with the radial components of the fields was the standard approach [6] for evaluating the beam shape coefficients in the Bromwich potential formulation of GLMT [17].

Returning to the main development presented in this paper, over the years much effort has gone into calculating $g^{TE}_{n,m}$ and $g^{TM}_{n,m}$ for a number of transversely localized beams, most notably for various models of a nominally Gaussian beam. There are, however, other possibilities for the incident beam that fall outside of the usual choices. Consider for example a linearly polarized plane wave component in the angular spectrum of an incident beam whose propagation direction is in the xz plane and having