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Convex Relaxation of Optimal Power Flow in Distribution Feeders with Embedded Solar Power

Alexander Hermann[a], Qiuwei Wu[a], Shaojun Huang[a*], Arne Hejde Nielsen[a]

[a]*Center for Electric Power and Energy, Technical Univeristy of Denmark, Elektrovej, Building 325, 2800 Lyngby, Denmark*

Abstract

There is an increasing interest in using Distributed Energy Resources (DER) directly coupled to end user distribution feeders. This poses an array of challenges because most of today's distribution feeders are designed for unidirectional power flow. Therefore when installing DERs such as solar panels with uncontrolled inverters, the upper limit of installable capacity is quickly reached in many of today's distribution feeders. This problem can often be mitigated by optimally controlling the voltage angles of inverters. However, the optimal power flow problem in its standard form is a large scale non-convex optimization problem, and thus can't be solved precisely and also is computationally heavy and intractable for large systems. This paper examines the use of a convex relaxation using Semi-definite programming to optimally control solar power inverters in a distribution grid in order to minimize the global line losses of the feeder. The mathematical model is presented in details. Further, case studies are completed with simulations involving a 15-bus radial distribution system. These simulations are run for 24 hour periods, with actual solar data and demand data.

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1. Nomenclature

p_i	Injected active power at bus *i*
q_i	Injected reactive power at bus *i*
p_{ij}	active power flow from node *i* to node *j*
q_{ij}	Reactive power flow from node *i* to node *j*
b_{ij}	Imaginary part of complex admittance matrix
g_{ij}	Real part of complex admittance matrix

* Corresponding author. Tel.: +45 45 25 34 95.
E-mail address: shuang@elektro.dtu.dk.

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$\underline{p_i}$	Lower limit of active power injection
$\overline{p_i}$	Upper limit of active power injection
$\lvert v_i \rvert$	Voltage magnitude at bus *i*
θ_i	Bus voltage angle at bus *i*
$v_i = \lvert v_i \rvert \angle \theta_i$	Bus voltage at bus *i*
$\boldsymbol{Y}$	Nodal Bus admittance matrix
Y_{ij}	The *ij*th entry of the Nodal bus admittance matrix
$Tr(\cdot)$	The trace operator
$diag(\cdot)$	The diagonal operator, returns diagonal of a matrix
$P_{d,i}$	Active power demand at bus *i*
$Q_{d,i}$	Reactive power demand at bus *i*
$\boldsymbol{I}_f$	Incidence Matrix from nodes
$\boldsymbol{I}_t$	Incidence Matrix to nodes

2. Introduction

There is going to be a radical change in the future distribution systems, due to upcoming needs and possibilities that are created by European and US initiatives such as EU FP7 Ideal Grid for all and US DOE Smart Grid[1]. This transformation from classical line drop compensated distribution grids to modern "Smart Grids" is enabled by the expected integration of modern communications systems, variable renewable energy sources and storage-capable loads such as batteries and grid connected electric vehicles.

Optimal control of the active/reactive power balance in Distributed Energy Resources (DER) and Demand Response Resources (DRR) such as battery storage [2], has become increasingly interesting with the possibility of them being installed and connected directly to the distribution grid(see e.g. [3]–[5]). In the past large arrangements of Wind Turbines (WTs) and Photo Voltaic panels (PVs) have been connected on a feeder of their own in order to ease the issues of over and under voltage and congestion. If DERs are connected directly to the Distribution network (DN), the Distribution System Operator (DSO) will most times require them to run at unity power factor or a fixed power factor, since they can interfere with DSO control measures such as on-load tap changing of the main feeder HV/MV transformers [6].

Also massive penetration of DERs in distribution networks is likely to have an effect on the Voltage limits in the feeder because resistance-to-reactance ratios r/x are such that voltage bus levels are quite sensitive to the active power injections [7], [8]. Therefore, hard limits on the amount of installable DERs are quickly reached due to over- and under-voltage concerns.

A DER on a feeder of a distribution system may even worsen the voltage levels in the system even if it is running at unity power factor [9]. Now, with the decreasing cost of PVs and other DERs such as battery storage, there is increasing interest in connecting them in large amounts to the end consumer distribution network. One of the most important issues of connecting large amounts of DERs to the DN are over- and under-voltage concerns [6], [10].

If DERs and DRRs are connected to the grid through controllable power converters, the opportunity arises to use to provide ancillary services to the grid [11]–[13]. One possibility is to use them to provide reactive power in order to aid the primary voltage control that is usually provided by tap-changing transformers. This will also help improve the lifetime of the on-load tap changing mechanisms, since they are designed for slow intra-hour load changes, and thus only operate a few times a day. When large amounts of renewable based DERs are connected to a feeder, the variations in load are much faster, and therefore a tap changer would have to operate several times an hour, which