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On the polarization behavior of diffraction by small elliptic aperture

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ABSTRACT

A theory on the diffraction of an elliptic Bethe–Bouwkamp aperture illuminated by a polarized plane-wave is established. The fictitious surface magnetic densities of charges and currents are rigorously represented by rewriting Bouwkamp's partial differential equations into vectorial expressions, and hence the electromagnetic field is described in a compact form. The polarization behaviors of both near-field diffraction and far-field radiation with respect to the incident light field are discussed. Novel phenomena owing to the geometry of elliptic aperture are demonstrated.

1. Introduction

As a fundamental property of light, diffraction has been studied for centuries ever since Huygens–Fresnel principle was established, which concludes that any primary wavefront can be viewed as the source of secondary disturbance. However, simple rules sometimes lead to quantities of confusing phenomenon, and just like a delicate gift from nature, diffraction problems always attract considerable attentions of scientists.

Analytical diffraction theories are early developed from the famous Helmholtz–Kirchhoff theorem [1]. The Kirchhoff's diffraction integral provides great applicability in macroscopic situations, but its boundary conditions are mathematically incorrect and inadequate for a smaller aperture when take into account the agreement with Maxwell's equations. Kottler's modification [2] introduces fictitious line charges at the edge represented by the contour integrals, which satisfies Maxwell's equations but still not rigorous to reveal the field in the vicinity of the aperture. Rigorous solutions ("wave equation plus boundary conditions" [3]), for diffraction problems are so difficult owing to the abstruse mathematics that few shapes of small aperture ($\ll \lambda$) can be properly dealt with. No such rigorous solutions are found until Sommerfeld's work [4] of a perfectly conducting semi-infinite screen in 1896. For circular disk or aperture, Bethe [5] revealed the diffraction of that case with his far-field superposition of dipoles which is a useful approximation for radiation problems. Bouwkamp [6] derived the correct solution of Bethe's near-field formulas and therefore Bethe–Bouwkamp (BB) model is proposed.

Diffraction problems of more complicated geometry are usually numerically simulated with various methods, e.g. multiple multipole (MMP) [7], finite difference time domain (FDTD) [8], and etc. [9,10]. Although numerical approaches are always of good flexibility for specific situations, their unavoidable disadvantage of poor phenomenal explanation limits their preferability when physical insight is required. Because the structure of near-field probe essentially resembles the situation of BB model [11], the description of near-field problems is undoubtedly one of the most straightforward applications of BB model, and plenty of amazing phenomenon, e.g., light-matter interaction [12] are to be theoretically clarified.

BB model has been recently confirmed [13] and compared [14,15] in experiments, some even involving oversize circular apertures (radius above $\lambda/5$). The polarization of a light beam diffracted by a BB aperture (or perhaps larger as subwavelength aperture) is promising for some novel applications, such as magnetic field analyzing [16] and field mapping [17]. Intensive efforts have been made for further comprehension and application of light with novel polarization, e.g. cylindrical vector beam, [18,19] which shows great potential in microscopic super resolution [20–22], optical communication [23], optical data storage [24], 3D nanofabrication [25], soliton diode [26] etc. As a matter of fact, arbitrarily spatial-variant vector beams have been designed and generated by devices like spatial light modulator [27], interferometer [28], and concepts as Poincaré sphere [29], or even the curl of polarization [30]. Meanwhile, near-field probes with novel-shaped apexes, elliptic [31] or bow tie-like [32], are intensively studied both theoretically and experimentally. Above all, a rigorous theory on

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the polarization behavior by a BB aperture with novel shapes and even with novel incident field is of sufficient necessity.

The purpose of this letter is to reveal the polarization behavior of an elliptic BB aperture with plane-wave incidence by giving the rigorous solutions of fictitious surface magnetic densities, near-field diffraction, and far-field radiation, which Bethe [5] has claimed a possible extension of his model. Both scalar [33] and vectorial [34] theories are recently proposed, the former based on Fourier transform while the latter deriving the field on the aperture with abundant rigorous results, to the best of our knowledge. Our vectorial results express near-field diffraction and far-field radiation in more easy-using formulas, where the polarization behaviors are demonstrated and explained. We find that scaling and rotation are applied into the diffracted electromagnetic field comparing to the incident one, with near-field electric polarization tilting towards the minor axis of the elliptic aperture. Furthermore, unexpected directions of the two dipoles in far-field radiation are derived and clarified, with potential applications for far-field description.

2. Theory

2.1. Backgrounds and settings

The theory reveals the polarization behavior of diffraction by an elliptic BB aperture located on a perfectly conducting screen (vanishing thickness small enough to be neglected), with whose size sufficiently small compared to the incident wavelength.

Right-handed rectangular coordinates is used to describe the model because of plane-wave incidence, while in terms of linear scaling (cf. Appendix) other coordinates are probably not as convenient as this one. The metallic (or perfectly conducting) screen S and the elliptic aperture A is located on the plane $z = 0$, on which A is treated as the ellipse center at the origin of the coordinate with a its semi-major axis and $0 \le \epsilon < 1$ its eccentricity.

A group of orthogonal basis as $(\boldsymbol{p}, \boldsymbol{s}, \boldsymbol{\kappa}^i)$ is created by rotating the three axes of this coordinates as $(\boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z)$ at a generalized angle, i.e.

$$\begin{bmatrix}\boldsymbol{p}\\ \boldsymbol{s}\\ \boldsymbol{\kappa}^i\end{bmatrix} = \begin{bmatrix}\cos\theta^i & 0 & -\sin\theta^i\\ 0 & 1 & 0\\ \sin\theta^i & 0 & \cos\theta^i\end{bmatrix}\begin{bmatrix}\cos\phi^i & \sin\phi^i & 0\\ -\sin\phi^i & \cos\phi^i & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\boldsymbol{e}_x\\ \boldsymbol{e}_y\\ \boldsymbol{e}_z\end{bmatrix} = \begin{bmatrix}\cos\theta^i\cos\phi^i & \cos\theta^i\sin\phi^i & -\sin\theta^i\\ -\sin\phi^i & \cos\phi^i & 0\\ \sin\theta^i\cos\phi^i & \sin\theta^i\sin\phi^i & \cos\theta^i\end{bmatrix}\begin{bmatrix}\boldsymbol{e}_x\\ \boldsymbol{e}_y\\ \boldsymbol{e}_z\end{bmatrix}, \tag{1}$$

where $\boldsymbol{\kappa}^i$ is viewed as the unit vector of the incident wave vector, and the other two as the unit vectors of P light and S light respectively. θ^i is made positive and light transmits from the left-hand side i.e. $z < 0$ to the right. Then the electromagnetic field, superscript i for "incident", as

$$\begin{cases}\boldsymbol{E}^i = (\cos\alpha\boldsymbol{p} + \sin\alpha\boldsymbol{s})\, e^{ik(\boldsymbol{\kappa}^i\cdot\rho)}\\ \sqrt{\mu/\varepsilon}\boldsymbol{H}^i = (-\sin\alpha\boldsymbol{p} + \cos\alpha\boldsymbol{s})\, e^{ik(\boldsymbol{\kappa}^i\cdot\rho)}\end{cases} \tag{2}$$

is established, with $\rho = x\boldsymbol{e}_x + y\boldsymbol{e}_y + z\boldsymbol{e}_z$ the spatial displacement and $e^{ik(\boldsymbol{\kappa}^i\cdot\rho)}$ the phase factor indicating an unit monochromatic plane wave.

Boundary conditions of BB model is specified [3] as

$$\begin{cases}E_n = E_n^i, \partial E_t/\partial n = \partial E_t^i/\partial n, \partial H_n/\partial n = \partial H_n^i/\partial n, H_t = H_t^i, \text{on } A\\ \partial E_n/\partial n = 0, E_t = 0, H_n = 0, \partial H_t/\partial n = 0, \text{on } S\end{cases} \tag{3}$$

where subscript n denotes "normal" and t denotes "tangential". Properties of odd functions are immediately found at the glance of E_t and H_n, while E_n and H_t are similar with even functions. Therefore, take $z = 0$ a fictitious mirror plane, $\boldsymbol{H}$ will be symmetric and $\boldsymbol{E}$ anti symmetric.

The unperturbed field of BB model, i.e. the electromagnetic field with A disappeared, is defined [3,5] as

$$\begin{cases}\boldsymbol{E}^0/2 = (\boldsymbol{n}\cdot\boldsymbol{E}^i)\,\boldsymbol{n}\\ \boldsymbol{H}^0/2 = (\boldsymbol{n}\times\boldsymbol{H}^i)\times\boldsymbol{n},\end{cases} \quad \text{on } z = 0, \tag{4}$$

which is valid only on the left-hand side of the screen, while on the other side $\boldsymbol{E}^0 = \boldsymbol{H}^0 = 0$ because of the vanishing thickness of the metallic screen.

The rigorous form of Babinet's principle [1] enables BB model to introduce fictitious surface densities of magnetic charges η and currents $\boldsymbol{K}$ on the aperture to represent the effect of real surface densities of electric ones on the metallic screen. Another group of boundary conditions on the right-hand side of S is quoted [35] as

$$\begin{cases}\boldsymbol{E} = \boldsymbol{E}^0/2 + \boldsymbol{n}\times\boldsymbol{K}\\ \boldsymbol{H} = \boldsymbol{H}^0/2 + (\eta/\mu)\,\boldsymbol{n},\end{cases} \tag{5}$$

which provides an approach to calculate the near-field diffraction. Furthermore, the electromagnetic field on the right-hand side is more concerned about, so all the following $\boldsymbol{E}$ and $\boldsymbol{H}$ will denote that field.

The vectorial potential function $\boldsymbol{F}$ in BB model is of vital importance owing to the fact that

$$\begin{cases}\boldsymbol{E} = \nabla\times\boldsymbol{F}\\ \boldsymbol{H} = \dfrac{1}{i\omega\mu}\nabla\times\nabla\times\boldsymbol{F}\\ \boldsymbol{F} = -\dfrac{1}{4\pi}\displaystyle\int_A (\boldsymbol{K}\mathfrak{g})\, d\Sigma.\end{cases} \tag{6}$$

where $\mathfrak{g} = e^{ikr}/r$ is a scalar Green's function and r denotes the distance between the field point and the source point. In addition, according to the continuity equation of charges and currents,

$$\nabla\cdot\boldsymbol{K} = ik\sqrt{\frac{\mu}{\varepsilon}}\left(\frac{\eta}{\mu}\right), \tag{7}$$

therefore once $\boldsymbol{K}$ is figured out, other variables can be calculated by Eqs. (6) and (7).

Far-field calculation is based on the vectorial diffraction formulas [5,35], written in SI units as

$$\begin{cases}\boldsymbol{E} = -\dfrac{e^{ik\rho}}{4\pi\rho}ik\boldsymbol{\kappa}\times\displaystyle\int_A \boldsymbol{K}\left(1 - ik\boldsymbol{\kappa}\cdot\rho'\right)d\Sigma\\ \boldsymbol{H} = \dfrac{e^{ik\rho}}{4\pi\rho}ik\sqrt{\dfrac{\varepsilon}{\mu}}\displaystyle\int_A \boldsymbol{K}\left(1 - ik\boldsymbol{\kappa}\cdot\rho'\right)d\Sigma\\ \quad -\dfrac{e^{ik\rho}}{4\pi\rho}ik\boldsymbol{\kappa}\displaystyle\int_A\left(\frac{\eta}{\mu}\right)\left(1 - ik\boldsymbol{\kappa}\cdot\rho'\right)d\Sigma,\end{cases} \tag{8}$$

where simplifications are applied on $\mathfrak{g}$, and $\boldsymbol{\kappa}$ denotes the unit vector of propagating direction which satisfies

$$\boldsymbol{\kappa} = \sin\theta\left(\cos\phi\boldsymbol{e}_x + \sin\phi\boldsymbol{e}_y\right) + \cos\theta\boldsymbol{e}_z \tag{9}$$

2.2. Vectorial expressions for derivatives of potential function

Boundary conditions expressed in $\boldsymbol{F}$ is quoted as [3]

$$\begin{cases}\partial F_y/\partial x - \partial F_x/\partial y = E_z^i\\ \left(\partial^2/\partial x^2 + \partial^2/\partial y^2\right)F_x = -\partial E_y^i/\partial z\\ \left(\partial^2/\partial x^2 + \partial^2/\partial y^2\right)F_y = \partial E_x^i/\partial z,\end{cases} \quad \text{on } A \tag{10}$$

where the wavenumber k is neglected because $ka \ll 1$, and $F_z = 0$ on the aperture in accordance with 3rd equation of (6). $\boldsymbol{F}$ is found complicated by substituting $\boldsymbol{E}^i$ from Eqs. (1) and (2) into Eqs. (10), however, an easier approach based on vectorial analyze as following will make it both straightforward for explanation and flexible for other kinds of incident light.

On one hand, E_z^i is simply handled by a Taylor series of in-plane displacement ρ, with $\boldsymbol{E}_0^i$ the value of $\boldsymbol{E}^i$ at the aperture center, written as

$$E_z^i = \boldsymbol{n}\cdot\boldsymbol{E}^i = (1 + \rho\cdot\nabla + \cdots)\left(\boldsymbol{n}\cdot\boldsymbol{E}_0^i\right) = \left(\boldsymbol{n}\cdot\boldsymbol{E}_0^i\right) + (\boldsymbol{n}\times\rho)\cdot\left[\boldsymbol{n}\times\nabla\left(\boldsymbol{n}\cdot\boldsymbol{E}^i\right)\right] + \cdots, \tag{11}$$

while on the other hand, the incident field satisfying Maxwell's equations,

$$\partial\boldsymbol{E}^i/\partial z = \boldsymbol{n}\cdot\nabla\boldsymbol{E}^i = -\boldsymbol{n}\times\left(ik\sqrt{\frac{\mu}{\varepsilon}}\boldsymbol{H}^i\right) + \nabla\left(\boldsymbol{n}\cdot\boldsymbol{E}^i\right), \tag{12}$$